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4. Title of the invention

POWER METER FOR AC ELECTRICAL SYSTEMS

5. Name of your agent *(if you have one)*

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POWER METER FOR AC ELECTRICAL SYSTEMS

F. Ghassemi

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July 1998

LIST OF CONTENT

POWER METER FOR AC ELECTRICAL SYSTEMS

The power meter is based on a new algorithm that measures active and reactive powers, apparent power and power factor for single and three phase ac systems. The present invention resides in the use of a modified instantaneous power calculation to derive a measurable quantity such as apparent power, power factor or energy consumption, wherein a reactive power component of the modified instantaneous power is defined as a dc imaginary component of the instantaneous power to satisfy power signal theories in both the time and frequency domain.

Power meters are used in industry to measure energy consumption of electrical energy users. There are four parameters that specify the quality and energy consumption of an electrical ac system, active power that when integrated in time determines the energy consumption, power factor, apparent power that determines the maximum demand and reactive power.

Measuring devices are manufactured according to different algorithms. The method that is used in majority of measuring devices is based on the standard power theory in ac circuits.

The growth of non-linear loads in power system has been the source of concerns for many years. One aspect that needs clarification and demands investigation is the definition of the basic electrical quantities such as apparent power, reactive power or power factor, when the signals are not sinusoidal. There is enormous literature on the subject that confirms the importance of the problem for the academia and industry. However, a convincing solution is still being sought. The concern is more aggravated when one notices that the basic power theory lacks firmness when a simple unbalanced three-phase system with sinusoidal waveform is considered.

Although the definition of active power, P, has widely been accepted, there are continuous controversy on the definition of apparent power, reactive power, and power factor in three phase systems with and without distorted waveforms. The problems have been well established and reported in many technical papers.

The theoretical basis of the working principle of the meter described in this application is based on a new power theory that is introduced, analysed and investigated in three articles, entitled as below that are attached to this application as Appendices A, B and C. The attached articles are an integral part of this application.

1- A NEW CONCEPT IN AC POWER THEORY
Part One: Single and Three Phase Systems

2- A NEW CONCEPT IN AC POWER THEORY
Part Two: The New Apparent Power and Power Factor in Terms of Symmetrical Components

3- A NEW CONCEPT IN AC POWER THEORY
Part Three: The New Apparent Power and Power Factor with Non-Sinusoidal Waveforms

A bibliography of previous researches and works is provided in Appendix D for further reference.

Figure 1 shows the schematic diagram of the meter. Figure 2 shows the flow chart for the algorithm.

The power meter whose working principle is based on the content of the attached articles calculates the apparent power, S by calculating the rms of the instantaneous powers $p_p(t)$ and $p_q(t)$. $p_p(t)$ is obtained from the product of the voltage and current signals whereas $p_q(t)$ is calculated from the product of the phase shifted voltage signal and current. The phase shift is $-90°$. The voltage signal is filtered prior to phase shifting to extract the fundamental frequency component. For three phase application the same procedure is carried out on the per phase basis. In this power meter the active and reactive powers are obtained by calculating the average values of the instantaneous power components $p_p(t)$ and $p_q(t)$ respectively. In the meter described, the power factor is calculated by dividing the average power by the apparent power.

The definition of the apparent power introduced and analysed in the attached articles and used in the power meter is uniform for single phase and balanced/unbalanced three phase systems with sinusoidal or non-sinusoidal waveforms. There is no need for different definitions or procedure when the system topology changes (please see articles parts 1, 2 and 3).

The power meter calculates the apparent power, active and reactive powers and the power factor in single-phase ac systems as defined and analysed in part one of the attached articles. The apparent power is defined as the 2-Norm (rms) of the modified instantaneous power as given in part one of the attached articles.

The power meter calculates the apparent power, active and reactive powers and the power factor in three-phase ac systems as defined and analysed in part one and two of the attached articles. The apparent power is defined as the 2-Norm (rms) of the modified instantaneous power as given in parts one and two of the attached articles.

The power meter calculates the apparent power that is independent of the reference point voltage when used in three phase systems. This implies that the measurement is not affected by the choice of the reference point in balanced or unbalanced three phase systems.

The power meter calculates the apparent power, active and reactive powers and the power factor in single-phase ac systems with non-sinusoidal waveforms as defined and analysed in part three of the attached articles.

The power meter calculates the apparent power, active and reactive powers and the power factor in three-phase ac systems with non-sinusoidal waveforms as defined and analysed in part three of the attached articles.

The power meter calculates the apparent power that is independent of the reference point voltage when used in three phase systems with non-sinusoidal waveforms. This implies that the measurement is not affected by the choice of the reference point in balanced or unbalanced three phase systems.

The power meter calculates the apparent power whose components are symmetrical apparent power S_s, unsymmetrical apparent power S_u and the distortion power D as defined in part three of the attached articles.

The distortion power D, defined in part three of the attached articles, that is a component of the measured apparent power by the meter described in this application is independent of the reference point voltage when three phase balanced/unbalanced systems are considered.

The algorithm is easily implemented using present day technology (please see attached articles parts 1, 2 and 3).

For single phase measurement it is required that one voltage input and one current input are used (please see attached articles parts 1 and 3).

Switch SW1 in Figure 1 is to select the total S and P and Q as described in attached articles or the suggested positive phase sequence (pps) voltage S, P and Q. This is provided in the meter as an option (please see attached articles parts 2 and 3).

Referring to Figure 1, the voltage signal is filtered to extract the fundamental frequency component (ffc). This is in line with the suggested method described in the attached articles (please see article part 3).

Referring to Figure 1, after filtering section three phase band limited voltages are fed to a section whose output is the positive phase sequence voltages. This is an option. The selector SW1 is used to select this option (please see articles parts 2 and 3).

Referring to Figure 1, the voltages are fed to the phase shifting section. This section produces the quadrature axis voltages as described in the attached articles (please see article part 1). This is an essential part of the algorithm.

Referring to Figure 1, the voltages and currents are multiplied on per phase basis to produce the instantaneous powers (IP), $p_p(t)$ and $p_q(t)$ (please see articles parts 1, 2 and 3).

Referring to Figure 1, the average values of $p_p(t)$ and $p_q(t)$ are extracted by the averager units whose outputs are the active and reactive powers respectively (please see articles parts 1 and 3).

Referring to Figure 1, $p_p(t)$ and $p_q(t)$ are also fed to two blocks that calculate the root mean square (rms) of $p_p(t)$ and $p_q(t)$ (please see articles parts 1 and 3).

Referring to Figure 1, the apparent power, AP, is calculated as shown in the attached articles parts 1, 2 and 3 using the outputs from the rms calculation units.

Referring to Figure 1, the power factor, pf, is calculated using the apparent power and active power as shown in the attached articles parts 1, 2 and 3.

Referring to Figure 1, the calculated parameters are fed to the display unit for gain adjustment and display.

APPENDIX A

A NEW CONCEPT IN AC POWER THEORY

Part One: Single and Three Phase Systems

F.Ghassemi

ABSTRACT

In this paper a new definition of apparent power in single and three phase systems is presented. The technique is based on the instantaneous power, which satisfies the law of conservation of energy. In this technique the reactive power is determined as a dc imaginary component of the instantaneous power. The apparent power is defined as the rms or 2-Norm of the instantaneous power. The technique satisfies the Plancherel's theorem both in time and frequency domain. It will be shown that the power factor can only be made to unity in a 3-phase balanced system if the new definition of apparent power is used.

1-INTRODUCTION

The definition of power in ac circuits has been a subject of research and investigation for many years. Although the definition of active power, P, has widely been accepted, there are continuous controversy on the definition of the apparent power, reactive power, and power factor in three phase systems and circuits with distorted waveforms [1, 2, 3]. The problems have been well established and reported in many technical papers [1, 2, 3, 4], but yet no acceptable solutions have been found.

In recent years, there have been reports on new methods for calculating and measuring the apparent power in three phase systems. However, all of these methods have their shortcomings and not totally accepted by the academia and industry.

The method reported and used in reference [3] and [4] utilises the equivalent voltage and current for three phase systems to calculate the apparent power. This has the main disadvantage that the outcome depends on the reference voltage. Also the physical bases of the core of the theory which is the assumption that reactive power is purely the magnitude of one of the oscillatory component of the instantaneous power (IP) and the apparent power is only related to the losses of the system, has not been totally accepted [5].

For example, by considering the figure in the discussion section of reference [3] (second discussor), one can see that the value of the apparent power as defined in the paper depends on the point of voltage measurement for the broken line. The apparent power would be different if the voltage is taken from the load side of the broken point to the case when the voltage is taken from the source side. Therefore, it is clearly evident that the value of the apparent power as defined in reference [3] and [4] may not be unique for a given load.

In this report, first a single-phase system is considered and the power theory in the time and frequency domain is investigated and a new technique is proposed. The proposed method for calculating the apparent power and hence power factor is then extended to three-phase systems. Throughout this work the focus has been on the instantaneous power, since the law of "conservation of energy" is satisfied by this power in an ac system [6]. Also, as has been discussed in reference [7], unlike the current calculation in the frequency domain, the present power theory in electrical engineering has the shortcoming that the time domain analysis does not fully agree with the frequency domain calculation [7]. This feature of the proposed method will be also addressed.

2-SINGLE PHASE SYSTEMS

Consider the circuit shown in Fig. 1. Steady state is considered only.

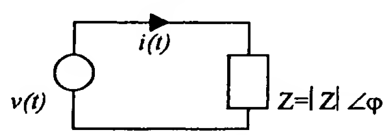

Fig 1-Single Phase Circuit

Equation (1) and (2) give the voltage and current respectively.

$$v(t)=\hat{V}\cos(\omega_1 t+\alpha) \tag{1}$$

$$i(t)=\hat{I}\cos(\omega_1 t+\beta) \tag{2}$$

It is clear that

$$\varphi=\angle Z=\alpha-\beta \tag{3}$$

The instantaneous power $p(t)$ is defined as follows:

$$p_p(t)=v(t)i(t)=\frac{\hat{V}\hat{I}}{2}\cos(\alpha-\beta)+\frac{\hat{V}\hat{I}}{2}\cos(2\omega_1 t+\alpha+\beta) \tag{4}$$

Equation (4) can be rearranged as follows:

$$p_P(t)=v(t)\,i(t)=\frac{\hat{V}\hat{I}}{2}\cos\varphi+\frac{\hat{V}\hat{I}}{2}\cos\varphi\cos(2\omega_1 t+2\beta)-\frac{\hat{V}\hat{I}}{2}\sin\varphi\sin(2\omega_1 t+2\beta) \tag{5}$$

The average value of Equation (5) is the power that is unidirectional. It is given by Equation (6).

$$P=\frac{1}{T}\int_0^T v(t)i(t)\mathrm{d}t=\frac{\hat{V}\hat{I}}{2}\cos\varphi \tag{6}$$

Where T is the period of the signal.
The standard active and reactive powers are defined as follows:

$$P\overset{\Delta}{=}\frac{\hat{V}\hat{I}}{2}\cos\varphi = \mathrm{VI}\cos\varphi \tag{7.a}$$

$$Q\overset{\Delta}{=}\frac{\hat{V}\hat{I}}{2}\sin\varphi = \mathrm{VI}\sin\varphi \tag{7.b}$$

Where V and I are the rms values of voltage and current respectively. Equation (5) can be rewritten in terms of P and Q as follows:

$$p_p(t)=v(t)i(t)=P+P\cos(2\omega_1 t+2\beta)-Q\sin(2\omega_1 t+2\beta) \tag{8}$$

Note that the reactive power is defined as the magnitude of one of the oscillatory components. The apparent power is then defined as follows.

$$S=VI=\sqrt{(VI\cos\varphi)^2+(VI\sin\varphi)^2}=\sqrt{P^2+Q^2} \tag{9}$$

It can be seen that the apparent power is defined as the product of the rms values of voltage and current.
Equation (9) defines a relationship between the apparent power and the active and reactive powers. These are the standard relationships that have been accepted and used for many years.
The Schwarz inequality states that for any two signals the following relationship holds:

$$\left(\left|\int_a^b x(t)y(t)\mathrm{d}t\right|\right)^2 \le \int_a^b |x(t)|^2\,\mathrm{d}t \cdot \int_a^b |y(t)|^2\,\mathrm{d}t \tag{10}$$

For generalisation of the relationship the magnitude of signals have been considered to include complex signals [8]. By inspection it can be seen that the left-hand side (L.H.S) is equal to the average power and right hand side (R.H.S) is equal to the product of the squared of the rms values of voltage and current. Thus:

$$(VI\cos\varphi)^2 \le V^2 I^2 \tag{11}$$

The inequality becomes equality when the angle between voltage and current, φ, is zero.
The reactive power considered as in (8) cannot be defined or obtained in frequency domain as discussed below.

3-POWER DEFINITION IN FREQUENCY DOMAIN

In two ways the frequency domain spectrum of power relationship, Equation (4), can be obtained.

1- By identifying the frequency components of voltage and current and performing convolution in frequency domain. This is equivalent to the multiplication of voltage and current in the time domain.
2- By calculating the instantaneous power, $p_p(t)$, and performing the Fourier transform.

Both methods must give the same result.

3.1-Calculation of Power from Frequency Spectrum of Voltage and Current

The Fourier transform of any signal $x(t)$ is defined as in Equation (12).

$$X(f)=\int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\mathrm{d}t \tag{12}$$

By performing the Fourier transform on (1) and (2), the frequency spectrum of voltage and current are determined.

$$V(f)=\int_{-\infty}^{+\infty} v(t)e^{-j\omega t}dt=\frac{1}{2}\left[\hat{V}e^{+j\alpha}\delta(f-f_1)+\hat{V}e^{-j\alpha}\delta(f+f_1)\right] \tag{13.a}$$

$$I(f)=\int_{-\infty}^{+\infty} i(t)e^{-j\omega t}dt=\frac{1}{2}\left[\hat{I}e^{+j\beta}\delta(f-f_1)+\hat{I}e^{-j\beta}\delta(f+f_1)\right] \tag{13.b}$$

Where δ is the Dirac function. The instantaneous power $p_p(t)$ is defined by Equation (4) as the product of the voltage and current. The same result can also be obtained in frequency domain by performing convolution of voltage and current spectrum, since the following relationship exists.

$$v(t)\,.\,i(t)\leftrightarrow V(f) * I(f) \tag{14.a}$$

And

$$v(t) * i(t)\leftrightarrow V(f)\,.\,I(f) \tag{14.b}$$

Where (.) and (*) denote multiplication and convolution respectively. The spectrum of voltage and current are shown in Fig 2.a and 2.b respectively.

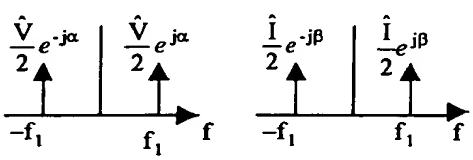

Fig 2.a Fig 2.b

Fig 2-Frequency Spectrum of Voltage and Current

2.a-Voltage, 2.b-Current

Note that the component at negative (-ve) frequency is the conjugate of that at positive (+ve) frequency. The –ve frequency does not have any physical meaning but it is the result of the mathematical analysis. It is important to note that both frequency components constitute the time domain signal in frequency domain, thus in order to be able to obtain the original signal in the time domain, both frequency components must be considered. The components of the instantaneous power can be obtained by convoluting the spectrum of voltage and current [9]. Thus:

$$P_p(f)=\int_{-\infty}^{+\infty} V(f)\,.\,I(f-f)\,df=\int_{-\infty}^{+\infty} I(f)\,.\,V(f-f)\,df \tag{15}$$

Where f is a dummy variable. Appendix A shows how (15) can be applied to calculate the frequency components of the instantaneous power. It can be seen that $P_p(f)$ has values at $-2f_1$, dc, and $2f_1$ as shown below:

$$P_P(-2f_1)=V(-f_1)I(-f_1)=\frac{\hat{V}\hat{I}}{4}e^{-j(\alpha+\beta)}=\frac{\hat{V}\hat{I}}{4}e^{-j\lambda} \tag{16.a}$$

Where: $\lambda=\alpha+\beta$ (16.b)

$$P_p(0)=V(-f_1)I(f_1)+V(f_1)I(-f_1)=\frac{\hat{V}\hat{I}}{4}e^{-j(\alpha-\beta)}+\frac{\hat{V}\hat{I}}{4}e^{+j(\alpha-\beta)}$$

$$P_p(0)=\frac{\hat{V}\hat{I}}{4}[e^{-j\varphi}+e^{+j\varphi}]=\frac{\hat{V}\hat{I}}{2}\cos\varphi=VI\cos\varphi=P \tag{17}$$

As can be seen, Equation (17) gives the average power that could be deduced from (6). Also, the frequency component at $2f_1$ is given by (18).

$$P_P(2f_1)=V(f_1)I(f_1)=\frac{\hat{V}\hat{I}}{4}e^{j(\alpha+\beta)}=\frac{\hat{V}\hat{I}}{4}e^{j\lambda} \tag{18}$$

The spectrum of the instantaneous power is shown in Fig 3.

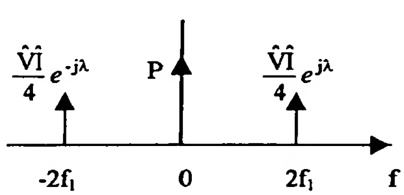

Fig 3- Frequency Spectrum of $P_p(f)$

As can be seen from Equation (17), the active power is that component of the instantaneous power that is unidirectional, i.e. dc. The same result can also be deduced from Equation (19.a) or (19.b).

$$P=[V(f_1)]^* I(f_1)+ V(f_1)I(f_1)]^* \qquad (19.a)$$

$$P= \sum_{f=-\infty}^{\infty} V(f)I^*(f) = \sum_{-f_1}^{f_1} V(f)\, I^*(f) \qquad (19.b)$$

As can be seen the frequency spectrum does not provide any information about the reactive power, Q.

3.2-Fourier Transform of the Instantaneous Power

If Fourier transform is applied on Equation (4), the same result as shown in Fig 3 is obtained.

4-DISCUSSION ON SPECTRUM ANALYSIS

The aforementioned analysis does not indicate that the reactive power as defined by (7.b) can be calculated in the frequency domain. Note that the Schwarz inequality suggests that if $\varphi \neq 0$, two sides of Equation (11) can never be equal. However, there is one situation when the equality is satisfied for any φ, and that is when the reactive power, Q, which is defined by Equation (7.b) is included in the equation. The analysis considered, which is the usual standard procedure adopted in electrical engineering does not have the capability to calculate the reactive power.

The power in any signal $x_p(t)$ may be found in terms of either a time or frequency description based on Parseval's relation if $x_p(t)$ is a square integrable function over one period [10]. A generalisation of this is Plancherel's relation for the product of two arbitrary periodic signal, $x_p(t)$ and $y_p(t)$. The Plancherel's relation is defined as in (20) [8, 10].

$$\frac{1}{T}\int_0^T x_p(t)[y_p(t)]^* dt = \sum_{-\infty}^{\infty} X(f)[Y(f)]^* \qquad (20)$$

L.H.S of Equation (20) is the average power that is given by (6). R.H.S of (20) is equal to what has been calculated by the convolution theorem in frequency domain that is described by (19.b). Thus the Plancherel's relation can be considered as a basis for defining the power in ac circuit. It is clear that if the normal procedure of power calculation is used, neither in time nor frequency domain, Q as defined by (7.b) can be determined.

In normal electrical engineering practice, complex power is considered that is defined by Equation (21).

$$\bar{S} = \bar{V}\bar{I}^* = P + jQ = |S|\angle\varphi \qquad (21)$$

Where $|S|$ is considered to be the apparent power in single and balanced three phase systems. Equation (21) is a frequency domain expression. However, it has been shown that the complex power cannot be deduced from the frequency spectrum of voltage and current. If R.H.S of (20) is modified, as shown in Equations (22), then Q can be deduced.

$$2\sum_{0}^{\infty} X(f)[Y(f)]^* = \frac{\hat{V}\hat{I}}{2} e^{j\varphi} = VI\cos\varphi + jVI\sin\varphi = P + jQ \qquad (22)$$

However, for this condition the Plancherel's (Parseval's) theorem is not satisfied and hence is not acceptable. In other words, the relationship (21) is not analytically valid.

5-TRADITIONAL METHOD FOR MEASURING Q

Traditionally, Q is measured by phase shifting the voltage signal by -90° and multiplying it by the current signal. Integration of the product over one period, which is effectively the average value, is the reactive power as given by (7.b) [11]. The procedure is given below.

$$p_q(t)=\hat{V}\cos(\omega_1 t+\alpha-\frac{\pi}{2}).\hat{I}\cos(\omega_1 t+\beta) \qquad (23.a)$$

$$p_q(t)=\frac{\hat{V}\hat{I}}{2}\sin\varphi+\frac{\hat{V}\hat{I}}{2}\sin(2\omega_1 t+\alpha+\beta) \qquad (23.b)$$

As can be seen, the average value of $p_q(t)$ is the reactive power, Q.

6-NEW APPARENT POWER

6.1-Modified Voltage Signal

As was shown in Section 5, the reactive power Q is obtained by assuming that it is an imaginary dc signal. In order to generalise the method, appropriate signal must be considered in the analysis that also satisfies all signal theories. Thus, if the voltage signal considered in the power calculation is modified as shown below then the reactive power Q can be calculated, as previously shown.

$$v_{mp}(t)=\hat{V}\cos(\omega_1 t+\alpha) \tag{1}$$

$$v_{mq}(t)=\hat{V}\cos(\omega_1 t+\alpha-\frac{\pi}{2})=\hat{V}\sin(\omega_1 t+\alpha) \tag{24}$$

Hence, the voltage signal that must be considered in the analysis is given below.

$$v_m(t)=v_{mp}(t)+jv_{mq}(t)=\hat{V}[\cos(\omega_1 t+\alpha)+j\sin(\omega_1 t+\alpha)]=\hat{V}e^{j(\omega_1 t+\alpha)} \tag{25}$$

Note that $v_{mq}(t)$ does not exist in the circuit but is physically obtainable as done in Q measurement. In order to keep in line with Plancherel's theorem the instantaneous power is defined as follows:

$$p(t)=v_m(t)[i(t)]^* \tag{26}$$

$$p(t)=\hat{V}e^{j(\omega_1 t+\alpha)}[\hat{I}\cos(\omega_1 t+\beta)]^* \tag{27}$$

Note that since the current is considered real, the conjugate is the same as signal itself. Using Cauchy relationship, the instantaneous power can be written as follows:

$$p(t)=\frac{\hat{V}\hat{I}}{2}e^{j\varphi}+\frac{\hat{V}\hat{I}}{2}e^{j(2\omega_1 t+\lambda)} \tag{28}$$

Where, $\varphi=\alpha-\beta$ and $\lambda=\alpha+\beta$.

As can be seen, $p(t)$ defined by (28) has dc components on the real and imaginary axes that are equal to P and Q respectively. Also, there exists a complex ac component. The spectrum of the IP is shown in Fig 4.

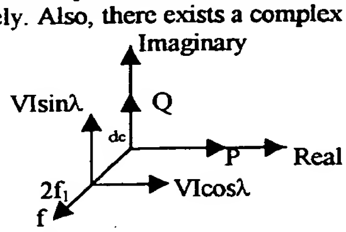

Fig 4- The Frequency Spectrum of the Instantaneous Power

Applying the Schwarz inequality to the modified signals, the following is obtained.

$$\left(\frac{\hat{V}\hat{I}}{2}\right)^2 \le \hat{V}^2\frac{\hat{I}^2}{2} \tag{29}$$

Note that if the voltage given in (25) is considered the rms of the signal is equal to its peak [9]. It can be seen that the relationship does not depend on the phase angle between the voltage and current. Hence for the modified signals, the Schwarz relationship can be written as follow.

$$\left(\sqrt{2}\left|\int_0^T v_m(t)i(t)dt\right|\right)^2 = \int_0^T |v_m(t)|^2 dt \cdot \int_0^T |i(t)|^2 dt \tag{30}$$

Equation (30) can be written as shown below.

$$\left(\sqrt{2}\sqrt{P^2+Q^2}\right)^2=\left(\sqrt{2}VI\right)^2=(V_m I)^2 \tag{31}$$

Where V_m denotes the rms value of the complex voltage signal. It can be seen that if the voltage signal is modified, the Schwarz relationship is always equality, provided √2 coefficient is considered.. Equation (31) shows the relationship between the product of the rms values of the voltage and current and the active and reactive powers.

The frequency spectrum of the complex voltage signal is given by Equation (32).

$$V(f)=\hat{V}e^{j\alpha}\delta(f-f_1)=V_m e^{j\alpha}\delta(f-f_1) \tag{32}$$

The spectrum of current is given by (13.b). Applying Equation (20) yields the active and reactive powers both in the time and frequency domain. (33.a) and (33.b) give the L.H.S and R.H.S of (20) respectively.

$$\frac{1}{T}\int_0^T v_m(t)\,i(t)dt=\frac{\hat{V}\hat{I}}{2}e^{j\varphi}=\frac{\hat{V}\hat{I}}{2}\cos\varphi+j\frac{\hat{V}\hat{I}}{2}\sin\varphi \tag{33.a}$$

$$\sum_{f=-\infty}^{\infty} V_m(f)[I(f)]^* = \sum_{f=-f_1}^{f_1} V_m(f)[I(f)]^*$$
$$=\hat{V}e^{j\alpha}\frac{\hat{I}}{2}e^{-j\beta}+0\cdot\frac{\hat{I}}{2}e^{j\beta}=\frac{\hat{V}\hat{I}}{2}e^{j\varphi}=\frac{\hat{V}\hat{I}}{2}\cos\varphi+j\frac{\hat{V}\hat{I}}{2}\sin\varphi \quad (33.b)$$

Thus, the Plancherel's relation is satisfied and at the same time the reactive power can be determined both in frequency and time domain and all signal theories are satisfied. As it is evident the reactive power is considered as an imaginary unidirectional power in this analysis.

6.2-Apparent Power as the NORM of the Instantaneous Power

Norm of any signal $x(t)$ is defined by Equation (34) [8, 12].

$$\|x\|_d = \left(\int_{-\infty}^{\infty} |x(t)|^d \,dt\right)^{\frac{1}{d}} \qquad \text{for} \qquad 1 \le d \le \infty \quad (34)$$

When $d=\infty$, the Norm is the largest magnitude the signal assumes. The 2-Norm (where $d=2$) is the rms value of the signal. It is a measure of the size of the signal [10, 12]. For any signal, the square of the 2-Norm is referred to as the energy (for energy signals) or power (for power signals) of the signal. It is clear that the periodic signals are power signals [9, 10] and $(\|x\|_2)^2$ gives the total power in the signal [8, 10,12]. Note that although the instantaneous power (IP) is not a physical signal [6], but it can be regarded as a signal. In fact the instantaneous power is a quantity for determining physical quantities. But it must also be emphasised that the law of the conservation of energy is satisfied by this quantity throughout the circuit.
The 2-Norm or rms of $p(t)$ is given by Equation (35). Note that since $p(t)$ is periodical (power signal) the limit of integration has been modified [12].

$$\|p\|_2 = \left(\underset{\lim T\to\infty}{\frac{1}{T}} \int_0^T |p(t)|^2 \,dt\right)^{\frac{1}{2}} \quad (35)$$

Substituting (28) into (35) yields.

$$\|p\|_2=\left[\frac{1}{T}\int_0^T\left[\left|\frac{\hat{V}\hat{I}}{2}e^{j\varphi}\right|^2+\left|\frac{\hat{V}\hat{I}}{2}e^{j(2\omega_1 t+\lambda)}\right|^2\right]dt\right]^{\frac{1}{2}} \quad (36.a)$$

$$\|p\|_2=\left[\left(\frac{\hat{V}\hat{I}}{2}\right)^2+\left(\frac{\hat{V}\hat{I}}{2}\right)^2\right]^{\frac{1}{2}}=\sqrt{2}\,\frac{\hat{V}\hat{I}}{2}=\sqrt{2}VI=\frac{\sqrt{2}}{\sqrt{2}}V_m\frac{\hat{I}}{\sqrt{2}}=V_m I \quad (36.b)$$

Equation (36.b) shows that the 2-Norm is equal to the product of the rms of the current and complex voltage or $\sqrt{2}$ times the rms of current and real voltage.
The apparent power is defined as the 2-Norm of the modified instantaneous power. The modified instantaneous power is the product of current and complex voltage. Thus, the relationship between the 2-Norm and different quantities for single-phase systems is given below.

$$S=\|p\|_2 \quad (37.a)$$
$$S=V_m I=\sqrt{2}VI=\sqrt{2}\sqrt{P^2+Q^2} \quad (37.b)$$

Note that the apparent power defined by Equation (37.a) has the dimension of volt-ampere.
The new definition of apparent power presented here leads to the product of the rms of current and voltage. It has been shown that in order to satisfy the rule of electrical engineering both in time and frequency domain, the definition of reactive power as the magnitude of one of the oscillatory components of $p(t)$ is not sufficient.

7-POWER FACTOR (pf) IN SINGLE PHASE SYSTEMS

Due to the uncertainty of definition of reactive power in 3-phase and distorted waveform systems the ratio of active power to the apparent power has been suggested by many researchers as the acceptable definition for pf [1, 2, 3]. With reference to the new definition for S, pf is given by Equation (38).

$$\mathrm{pf} = \frac{P}{S} \tag{38}$$

Where $P = VI\cos\varphi$
$S = V_m I$

$V_m = \hat{V}$ magnitude of the voltage signal (rms of the complex voltage signal)
I= rms of the current signal

With reference to Equation (37.b), in single-phase systems the pf can be written as (39).

$$\mathrm{pf} = \frac{P}{S} = \frac{P}{\sqrt{2}\,VI} = \frac{P}{\sqrt{2}\sqrt{P^2+Q^2}} = \frac{\cos\varphi}{\sqrt{2}} \approx 0.707\cos\varphi \tag{39}$$

It can be seen that pf cannot be greater that 0.707 in single-phase systems. The maximum pf occurs when the voltage and current are in phase ($\cos\varphi=1$), i.e. pure resistive load. This reduction in pf from standard unity pf for resistive load is due to the ac component of *p(t)*. This component exists even if all the reactive power (the component of *p(t)* on the dc imaginary axis) is compensated by reactive passive elements. The sign of Q determines the nature of the reactive power.

8-IMPLEMENTATION OF THE ALGORITM IN A MEASURING DEVICE

As has been previously mentioned, the 2-Norm of a signal is its rms value, which is equivalent to a dc signal with the same power [10]. Appendix B illustrates how the rms of *p(t)* can be calculated and that it is equal to Equation (40)

$$\begin{aligned} S = \mathrm{rms}[p(t)] &= \left\{\left[\mathrm{rms}\left[v_p(t)\,i(t)\right]\right]^2 + \left[\mathrm{rms}\left[v_q(t)\,i(t)\right]\right]^2\right\}^{\frac{1}{2}} \\ &= \sqrt{2P^2+2Q^2} = \sqrt{2}\sqrt{P^2+Q^2} \end{aligned} \tag{40}$$

Equation (40) is the same as (37.b). Note that the apparent power defined by (40) is not measured by the product of the rms of voltage and current but from the instantaneous power *p(t)*.

9-THREE PHASE SYSTEMS

The method proposed here for measuring S in 3-phase systems is the extension of the algorithm, which was presented for single-phase systems. It is believed that any method that is considered for calculating the apparent power should be based on a quantity that agrees with the law of energy conservation and does not violate any rule of basic electrical engineering. In any 3-phase system the sum of the instantaneous powers is zero. Thus;

$$\sum_{k=a,b,c} p_{gk}(t) + \sum_{k=a,b,c} p_{Lk}(t) + \sum_{k=a,b,c} \Delta p_k(t) = 0 \tag{41}$$

Where: $p_g(t)$= Instantaneous power at the source
$p_L(t)$= Instantaneous power at the load
$\Delta p(t)$= Instantaneous power loss

It has already been suggested that pf, which is a measure of the quality of loads, is calculated using the active and apparent powers [1, 2, 3]. The main question has been what the apparent power is in a 3-phase system. It is well known that in a 3-phase system with linear, unbalanced pure resistive loads there exists a difference between the active and apparent power, without any oscillation of power between load and source [2, 5]. Hence for such loads pf is not unity. The following section presents the new definition of S in 3-phase systems.

10-APPARENT POWER CALCULATED FROM 3-PHASE INSTANTANEOUS POWER

Consider a 3-phase source feeding a 3-phase load as shown in Fig 5.

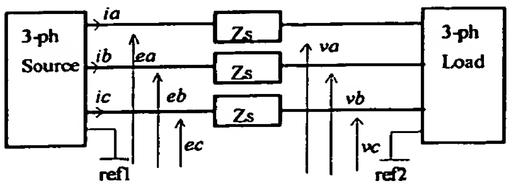

Fig 5-3-phase Source and Load

The analysis will be presented for the source and load separately.

10.1- Apparent Power and pf at the Source

Assume that the voltages and currents at the source are given by Equations (42.a) and (42.b).

$$\left.\begin{aligned} e_a(t)&=\hat{E}\cos(\omega_1 t)\\ e_b(t)&=\hat{E}\cos(\omega_1 t-\frac{2\pi}{3})\\ e_c(t)&=\hat{E}\cos(\omega_1 t+\frac{2\pi}{3}) \end{aligned}\right\} \quad (42.a)$$

$$\left.\begin{aligned} i_a(t)&=\hat{I}_a\cos(\omega_1 t+\beta_a)\\ i_b(t)&=\hat{I}_b\cos(\omega_1 t+\beta_b)\\ i_c(t)&=\hat{I}_c\cos(\omega_1 t+\beta_c) \end{aligned}\right\} \quad (42.b)$$

Where a, b, c denote the phase numbers. All phase angles are measured with respect to (w.r.t.) the phase "a" voltage of the generator. In order to consider a general case, the currents have been considered to be unbalanced. As has been previously presented, the voltage signals are modified. Thus the complex voltages are given below.

$$\left.\begin{aligned} e_{ma}(t)&=\hat{E}e^{j\omega_1 t}\\ e_{mb}(t)&=\hat{E}e^{j(\omega_1 t-\frac{2\pi}{3})}\\ e_{mc}(t)&=\hat{E}e^{j(\omega_1 t+\frac{2\pi}{3})} \end{aligned}\right\} \quad (43)$$

Per phase and total instantaneous power are given below.

$$\left.\begin{aligned} p_a(t)&=e_{ma}(t)\,i_a^*(t)=\frac{\hat{E}\hat{I}_a}{2}e^{j\varphi_a}+\frac{\hat{E}\hat{I}_a}{2}e^{j(2\omega_1 t+\lambda_a)}\\ p_b(t)&=e_{mb}(t)\,i_b^*(t)=\frac{\hat{E}\hat{I}_b}{2}e^{j\varphi_b}+\frac{\hat{E}\hat{I}_b}{2}e^{j(2\omega_1 t+\lambda_b)}\\ p_c(t)&=e_{mc}(t)\,i_c^*(t)=\frac{\hat{E}\hat{I}_c}{2}e^{j\varphi_c}+\frac{\hat{E}\hat{I}_c}{2}e^{j(2\omega_1 t+\lambda_c)} \end{aligned}\right\} \quad (44)$$

$$p(t)=p_a(t)+p_b(t)+p_c(t) \quad (45)$$

Where: $\varphi_a=-\beta_a$ $\qquad$ $\lambda_a=\beta_a$

$\varphi_b=-120^\circ-\beta_b$ $\qquad$ $\lambda_b=-120^\circ+\beta_b$

$\varphi_c=120^\circ-\beta_c$ $\qquad$ $\lambda_c=120^\circ+\beta_c$

Substituting (44) into (45) yields:

APPENDIX A

$$p(t)=\frac{\hat{E}\hat{I}_a}{2}e^{j\varphi_a}+\frac{\hat{E}\hat{I}_b}{2}e^{j\varphi_b}+\frac{\hat{E}\hat{I}_c}{2}e^{j\varphi_c}+\left[\frac{\hat{E}\hat{I}_a}{2}e^{j\lambda_a}+\frac{\hat{E}\hat{I}_b}{2}e^{j\lambda_b}+\frac{\hat{E}\hat{I}_c}{2}e^{j\lambda_c}\right]e^{j2\omega_1 t} \tag{46}$$

It is clear that the average value of (46) is the total active and reactive powers as indicated below.

$$\frac{1}{T}\int_0^T p(t)dt=\sum_{k=a,b,c}P_k+j\sum_{k=a,b,c}Q_k \tag{47}$$

Note that (47) represents only the dc part of the instantaneous power. The counterpart relation of (47) is obtained in frequency domain using the convolution (or Plancherel's relation).

$$\sum_{k=a,b,c}P_k+j\sum_{k=a,b,c}Q_k=\sum_{k=a,b,c}\left(\sum_{f=-f_1}^{f_1}E_{mk}(f)\,I_k^*(f)\right) \tag{48.a}$$

$$=(P_a+P_b+P_c)+j\,(Q_a+Q_b+Q_c) \tag{48.b}$$

Where E_{ma}, E_{mb}, E_{mc}, are frequency components of the complex voltages.
By the new definition, the rms or 2-Norm of *p(t)* is the apparent power. Substituting Equation (46) into (37.a) yields.

$$S=\left\{\left|(P_a+P_b+P_c)+j(Q_a+Q_b+Q_c\right|^2+\left|\left(\frac{\hat{E}\hat{I}_a}{2}\cos\lambda_a+\frac{\hat{E}\hat{I}_b}{2}\cos\lambda_b+\frac{\hat{E}\hat{I}_c}{2}\cos\lambda_c\right)+j\left(\frac{\hat{E}\hat{I}_a}{2}\sin\lambda_a+\frac{\hat{E}\hat{I}_b}{2}\sin\lambda_b+\frac{\hat{E}\hat{I}_c}{2}\sin\lambda_c\right)\right|^2\right\}^{0.5} \tag{49}$$

Equation (49) can be written as follow:

$$S=\left[S_s^2+S_u^2\right]^{0.5} \tag{50}$$

Where

$$S_s=\sqrt{(P_a+P_b+P_c)^2+(Q_a+Q_b+Q_c)^2}$$
$$=E\sqrt{I_a^2+I_b^2+I_c^2+2I_aI_b\cos(\varphi_a-\varphi_b)+2I_bI_c\cos(\varphi_b-\varphi_c)+2I_cI_a\cos(\varphi_c-\varphi_a)} \tag{51.a}$$
$$S_u=E\sqrt{I_a^2+I_b^2+I_c^2+2I_aI_b\cos(\lambda_a-\lambda_b)+2I_bI_c\cos(\lambda_b-\lambda_c)+2I_cI_a\cos(\lambda_c-\lambda_a)} \tag{51.b}$$

E is the rms of the real voltage signal at the generator. S_s and S_u are called the symmetrical and unsymmetrical apparent powers respectively, since as it will be shown S_u will be non-zero only if the currents are unbalanced. In a special case when the system is completely balanced, then S_s will be as shown below.

$$S_s=\sqrt{(P_a+P_b+P_c)^2+(Q_a+Q_b+Q_c)^2}=3\sqrt{P^2+Q^2}=3EI \tag{52}$$

For balanced system, $\beta_a=\beta_b=\beta_c=\beta$ and $I_a=I_b=I_c=I$ thus;
$\lambda_a-\lambda_b=\beta+120^\circ-\beta=120^\circ$
$\lambda_b-\lambda_c=-120^\circ+\beta-120^\circ-\beta=120^\circ$
$\lambda_c-\lambda_a=120^\circ+\beta-\beta=120^\circ$
Hence;

$$S_u=E\sqrt{I^2+I^2+I^2+2I\,I\cos(120)+2I\,I\cos(120)+2I\,I\cos(120)}=0 \tag{53}$$

This result is also evident from (46) where its ac part becomes zero as shown below.

$$\text{ac part of } p(t)=\frac{\hat{E}\hat{I}}{2}\left[e^{j\beta}+e^{j(-120+\beta)}+e^{j(120+\beta)}\right]=0 \tag{54}$$

Thus, in a balanced 3-phase system the ac part of the instantaneous power is zero and it consists of only dc components that is given by (52). This is contrary to the single-phase systems.
The power factor that is the ratio of the active power to apparent power can be made equal to unity for a balanced system if appropriate compensation is employed in the circuit, as this reduces the value of *p(t)* on the dc imaginary axis to zero and only dc real component is present. Also, it can be said that if there is an ac component on *p(t)* then it

means that there is unbalanced load present in the system. The unity pf is achievable in a 3-phase system by first balancing the load, using the techniques described in [11] and then compensating the reactive power, which is the dc component on imaginary axis.

10.2- Apparent Power and pf at the Load

The procedure for calculating the apparent power at the load is the same as that at the generator. The complex voltages at the load are defined as shown below;

$$\left.\begin{aligned} v_{ma} &= \hat{V}_a e^{j(\omega_1 t+\alpha_a)} \\ v_{mb} &= \hat{V}_b e^{j(\omega_1 t+\alpha_b)} \\ v_{mc} &= \hat{V}_c e^{j(\omega_1 t+\alpha_c)} \end{aligned}\right\} \tag{55}$$

The current signals are given by Equation (42.b). it is clear that if the currents are not balanced, due to the voltage drop on the source impedance, the voltage at the load cannot be balanced. The instantaneous power of the load is given below;

$$p(t)=\frac{\hat{V}_a\hat{I}_a}{2}e^{j\varphi_a}+\frac{\hat{V}_b\hat{I}_b}{2}e^{j\varphi_b}+\frac{\hat{V}_c\hat{I}_c}{2}e^{j\varphi_c}+\left[\frac{\hat{V}_a\hat{I}_a}{2}e^{j\lambda_a}+\frac{\hat{V}_b\hat{I}_b}{2}e^{j\lambda_b}+\frac{\hat{V}_c\hat{I}_c}{2}e^{j\lambda_c}\right]e^{j2\omega_1 t} \tag{56}$$

Where: $\varphi_a=\alpha_a-\beta_a$ $\quad\lambda_a=\alpha_a+\beta_a$

$\varphi_b=\alpha_b-\beta_b$ $\quad\lambda_b=\alpha_b+\beta_b$

$\varphi_c=\alpha_c-\beta_c$ $\quad\lambda_c=\alpha_c+\beta_c$

The apparent power at the load is the 2-Norm of the total instantaneous power. S at the load is given by (50), where the unbalance and balance components of the apparent power are given by (57) and (58).

$$\begin{aligned} S_s = \{&V_a^2I_a^2+V_b^2I_b^2+V_c^2I_c^2+2V_aV_bI_aI_b\cos(\varphi_a-\varphi_b)+ \\ &2V_bV_cI_bI_c\cos(\varphi_b-\varphi_c)+2V_cV_aI_cI_a\cos(\varphi_c-\varphi_a)\}^{0.5} \end{aligned} \tag{57}$$

$$\begin{aligned} S_u = \{&V_a^2I_a^2+V_b^2I_b^2+V_c^2I_c^2+2V_aV_bI_aI_b\cos(\lambda_a-\lambda_b)+ \\ &2V_bV_cI_bI_c\cos(\lambda_b-\lambda_c)+2V_cV_aI_cI_a\cos(\lambda_c-\lambda_a)\}^{0.5} \end{aligned} \tag{58}$$

It can be shown that if the system is balanced the apparent power is given by Equation (59);

$$S=3VI \tag{59}$$

11-MEASUREMENT OF THE NEW S IN 3-PHASE SYSTEMS

The algorithm for 3-phase systems can be implemented using the same technique as that described in Appendix B. The rms of 3-phase $p(t)$ is given below;

$$\mathrm{rms}[p(t)]=\left\{\left[\mathrm{rms}\left(\sum_{k=a,b,c} v_{pk}(t)i_k(t)\right)\right]^2+\left[\mathrm{rms}\left(\sum_{k=a,b,c} v_{qk}(t)i_k(t)\right)\right]^2\right\}^{\frac{1}{2}} \tag{60}$$

Equation (60) can be implemented on a measuring device.

12-CONCLUSION

It was shown that the present power theory cannot be used to determine the reactive power in time and frequency domain. The Plancherel's relation is not satisfied by the present power theory. A technique was proposed to determine the reactive power that satisfies signal theories in both time and frequency domain. The technique is based on the modification of the voltage signal by including a quadrature axis voltage. This component is obtained by delaying the voltage signal by 90°. The new apparent power is defined as the rms or 2-Norm of the instantaneous power that satisfies the law of conservation of energy in a circuit.

It was shown that if the new apparent power is considered the power factor defined, as the ratio of the active to apparent powers, cannot be unity in single-phase systems even if the voltage and current are in-phase. Unity power factor is only obtainable in 3-phase balanced systems. It was shown that for this condition the ac component of the instantaneous power is zero.

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14-APPENDICES

14.1-APPENDIX A

Applying Equation (15) on voltage and current spectrum yields:

$$P_p(-2f_1)=V(-2f_1)I(-2f_1+2f_1)+V(-f_1)I(-2f_1+f_1)+V(0)I(-2f_1)+V(f_1)I(-2f_1-f_1)$$

$$P_p(-2f_1)=V(-f_1)I(-f_1)=\frac{\hat{V}\hat{I}}{4}e^{-j(\alpha+\beta)}=\frac{\hat{V}\hat{I}}{4}e^{-j\lambda} \quad \text{(A1.a)}$$

Where: $\lambda=\alpha+\beta$ (A1.b)

And;

$$P_p(-f_1)=V(-3f_1)I(-f_1+3f_1)+V(-2f_1)I(-f_1+2f_1)+V(-f)I(-f_1+f_1)+V(0)I(-f_1)+ V(f_1)I(-f_1-f_1)+V(2f_1)I(-f_1-2f_1)+V(3f_1)I(-f_1-3f_1)=0 \quad \text{(A2)}$$

$$P_p(0)=V(-2f_1)I(0+2f_1)+V(-f_1)I(0+f_1)+V(0)I(0-0)+V(f_1)I(0-f_1)+V(2f_1)I(0-2f_1)$$

$$P_p(0)=V(-f_1)I(+f_1)+V(f_1)I(-f_1)=\frac{\hat{V}\hat{I}}{4}e^{-j(\alpha-\beta)}+\frac{\hat{V}\hat{I}}{4}e^{+j(\alpha-\beta)}=\frac{\hat{V}\hat{I}}{2}\cos\varphi \quad \text{(A3)}$$

By symmetry $P_p(f_1)=0$

$$P_p(2f_1)=V(-f_1)I(2f_1+f_1)+V(0)I(2f_1-0)+V(f_1)I(2f_1-f_1)=V(f_1)I(f_1)$$

$$P_p(2f_1)=V(f_1)I(f_1)=\frac{\hat{V}\hat{I}}{4}e^{j\lambda} \quad \text{(A4)}$$

Note that the frequency components lower than $-2f_1$ and higher than $2f_1$ are all zero.

14.2-APPENDIX B

The rms of $p(t)$ is given by B1.

$$\text{rms}[p(t)]=\text{rms}\{v_m(t)\,i(t)\}=\text{rms}\{\hat{V}e^{j(\omega_1 t+\alpha)}\,\hat{I}\cos(\omega_1 t+\beta)\} \quad \text{(B1)}$$

By expanding the voltage signal, Equation (B1) can be rearranged as follows:

$$\text{rms}[p(t)]=\left\{\left[\text{rms}\left[v_p(t)\,i(t)\right]\right]^2+\left[\text{rms}\left[v_q(t)\,i(t)\right]\right]^2\right\}^{\frac{1}{2}} \quad \text{(B2)}$$

Where:

$$\text{rms}\left[v_p(t)\,i(t)\right]=\text{rms}\left[VI\cos(\omega_1 t+\alpha)\cos(\omega_1 t+\beta)\right] =\sqrt{P^2+\frac{1}{2}P^2+\frac{1}{2}Q^2} \quad \text{(B3)}$$

$$\text{rms}\left[v_q(t)\,i(t)\right]=\text{rms}\left[VI\sin(\omega_1 t+\alpha)\cos(\omega_1 t+\beta)\right] =\sqrt{Q^2+\frac{1}{2}Q^2+\frac{1}{2}P^2} \quad \text{(B4)}$$

Thus;

$$\text{rms}\left[p(t)\right]=\sqrt{2P^2+2Q^2}=\sqrt{2}\sqrt{P^2+Q^2} \quad \text{(B5)}$$

APPENDIX B

A NEW CONCEPT IN AC POWER THEORY

Part Two: The New Apparent Power and Power Factor in Terms of Symmetrical Components

F.Ghassemi

ABSTRACT The new apparent power defined as the 2-Norm of the instantaneous power is investigated using the symmetrical components. It will be shown that the instantaneous power consists of different terms that are the result of the product of different components. The effects of each component on the apparent power and the power factor are investigated. It will be shown that the unbalanced loads convert some of their positive phase sequence active and reactive powers into other components but the same nature power and feed them back to the circuit. The same result is also obtained for the power components that are the result of the product of unlike components.

1-INTRODUCTION

Three phase systems can be analysed by the virtue of the symmetrical components. If the transmission system can be considered symmetrical, i.e. equal line impedances, the voltages and currents can be resolved into symmetrical components.

In part one of this report a new definition for the apparent power (AP) based on the modified instantaneous power (IP) was proposed. The AP was defined as the rms of the modified IP. The analysis was based on the phase quantities of 3-phase systems. In order to investigate the generalisation of the approach, outlined in part one of this report, the analysis is carried out here for 3-phase systems using the symmetrical components. This enables one to observe the influence of different components, namely positive, negative and zero phase sequence of voltages and currents on power components and power factor (pf).

Reference [1] used symmetrical components to obtain an expression for the AP in terms of rms values of voltages and currents. Power factor (pf) was then defined as the ratio of positive phase sequence (pps) power and the defined AP [1]. In the following sections, pf in terms of the symmetrical components will be investigated and a new pf will be introduced.

2-SYMMETRICAL COMPONENTS REPRESENTATION OF THE NEW AP

Consider a 3-phase ideal source feeding a 3-phase load via a transmission system with impedance Z_s. The source and load terminals will be considered separately.

2.1-The AP and pf at the Source

The voltages at the source can be defined as follows;

$$e_k^+=\hat{E}\cos(\omega_1 t+\alpha_k^+)\Big|_{k=a,b,c} \tag{1}$$

Where a, b, c denote the phase numbers. The magnitudes are phase quantities. Also;

$\alpha^+_a=0\ \alpha^+_b=-120,\ \alpha^+_c=120.$ (2)

Since 3-phase sources can only generate pps voltages the negative phase sequence (nps) and zero phase sequence (zps) voltages are all zero.

For each phase a voltage signal delayed by $-90°$ is formed and then as discussed in [2] the voltages are modified as shown below.

$$e_{qk}^+=\hat{E}\cos(\omega_1 t+\alpha_k^+-90^\circ)\Big|_{k=a,b,c} \tag{3}$$

$$e_{mk}^+=\hat{E}e^{j(\omega_1 t+\alpha_k^+)}\Big|_{k=a,b,c} \tag{4}$$

Equ. (4) defines the complex voltage signals that are used in the analysis. The phase angles are as given by (2) [2].

The phase voltages and currents can be resolved into symmetrical components at any point in the circuit [1]. Hence if the currents at the source are defined as (5) then using Cauchy relation, their symmetrical components representation will be as shown in (6).

$$i_k(t)=\hat{I}_k\cos(\omega_1 t+\beta_k)\Big|_{k=a,b,c} \tag{5}$$

$$i_k^r(t)=\left.\left.\frac{\hat{I}^r}{2}\left[e^{j(\omega_1 t+\beta_k^r)}+e^{-j(\omega_1 t+\beta_k^r)}\right]\right|^{r=+,-,o}\right|_{k=a,b,c} \tag{6}$$

Where:

$$\left.\begin{array}{l}\beta_a^+=\beta^+,\ \beta_b^+=\beta^+-120^\circ,\ \beta_c^+=\beta^++120^\circ\\ \beta_a^-=\beta^-,\ \beta_b^-=\beta^-+120^\circ,\ \beta_c^-=\beta^--120^\circ\\ \beta_a^o=\beta^o,\ \beta_b^o=\beta^o,\ \beta_c^o=\beta^o\end{array}\right\} \tag{7}$$

And $\hat{I}^+$, $\hat{I}^-$, $\hat{I}^o$ denote the magnitude of pps, nps and zps currents respectively. Also β^+, β^- and β^o are phase angle of pps, nps and zps currents with respect to (w.r.t) the phase "a" voltage of the source.
The per phase and total instantaneous powers (IP) at the source are defined by (8.a) and (8.b) respectively.

$$p_k(t)=e_{mk}(t)i_k^*(t)\Big|_{k=a,b,c}=e_{mk}^+(t)\left(i_k^+(t)+i_k^-(t)+i_k^o(t)\right)^*\Big|_{k=a,b,c} \tag{8.a}$$

$$p(t)=\sum_{k=a,b,c}p_k(t)=\sum_{k=a,b,c}e_{mk}^+(t)i_k^{+*}+\sum_{k=a,b,c}e_{mk}^+(t)i_k^{-*}+\sum_{k=a,b,c}e_{mk}^+(t)i_k^{o*} \tag{8.b}$$

The sum of the product of the pps voltages and zps currents are zero since the pps voltages are balanced and zps currents are all in-phase as shown below.

$$\overset{+}{p}{}^{o}(t)=\sum_{k=a,b,c}e_{mk}^+(t)i_k^{o*}(t)=\frac{\hat{E}\hat{I}^o}{2}\left[e^{-j(\omega_1 t+\beta^o)}+e^{j(\omega_1 t+\beta^o)}\right]\cdot\left[e^{j(\omega_1 t)}+e^{j(\omega_1 t-120)}+e^{j(\omega_1 t+120)}\right]=0 \tag{9}$$

This is physically acceptable since, theoretically, due to the physical placement of 3-phase windings in generators the zps current cannot produce any armature reaction in the machines.
Other components of the IP are found by considering the appropriate current components in (6). Thus;

$$p^+(t)=\sum_{k=a,b,c}e_{mk}^+(t)i_k^+(t)=3EI^+e^{j\varphi^+} \tag{10.a}$$

$$p^\pm(t)=\sum_{k=a,b,c}e_{mk}^+(t)i_k^-(t)=3EI^-e^{j(2\omega_1 t+\lambda^\pm)} \tag{10.b}$$

Where, $\varphi^+=-\beta^+$, $\lambda^\pm=\beta^-$ and E, I^+ and I^- are the rms of the terminal voltage, pps and nps currents respectively. Note that (10.a) is a dc whereas (10.b) is an ac signal. Note also that since the currents are real signals their conjugates are the same as the signal itself.
The total IP is given below;

$$p(t)=3EI^+e^{j\varphi^+}+3EI^-e^{j\lambda^\pm}e^{j(2\omega_1 t)} \tag{11}$$

It can be seen that the average value of (11) that is the same as (41) in [2], is the total active and reactive powers that are supplied by the source to the circuit. Also note that the 3-phase generator can only produce active and reactive powers in pps circuit. It is clear that if the system is balanced the nps current is zero and hence the IP consists of a dc term as explained in [2].
The apparent power (AP) was defined in [2] as the 2-Norm of the IP. Thus;

$$S=\|p\|_2=\left(\lim_{T\to\infty}\frac{1}{T}\int_0^T|p(t)|^2\,dt\right)^{0.5} \tag{12}$$

Substituting (11) into (12) yields:

$$S=\left\{(3EI^+)^2+(3EI^-)^2\right\}^{\frac{1}{2}}=3E\sqrt{I^{+^2}+I^{-^2}} \tag{13}$$

The balanced and unbalanced AP in terms of symmetrical components are defined as below;

$$S_s = 3EI^+ \tag{14.a}$$

$$S_u = 3EI^- \tag{14.b}$$

Equation (14.a) and (14.b) are equal to (50.a) and (50.b) in [2] respectively where S_s and S_u were defined in terms of the phase quantities.

The power factor (pf) of the source is defined as the ratio of the active power to the AP. Thus;

$$pf = \frac{P}{S} = \frac{P^+}{S} = \frac{I^+ \cos\varphi^+}{\sqrt{I^{+^2} + I^{-^2}}} \tag{15}$$

Note that since the machine can only produce pps active power, the pf is obtained in terms of P^+ only. Also note that since the pps circuit is balanced then any reference that is taken for the voltage measurement has zero voltage and thus the product of the rms of the pps voltage to the pps and zps currents is independent of the reference.

2.2-The AP and pf at the Load

2.2.1-IP Components at the Load

Equation (5) or (6) gives the currents in the circuit. Depending on the grounding of the system, the zps current may or may not be present in the line. In general, the pps, nps and zps voltage drops on the line impedances are given below:

$$\Delta v^r(t) = (R_s^r + L_s^r \frac{d}{dt}) i^r(t) \Big|^{r=+,-,o} \tag{16}$$

Where R_s and L_s are the line sequence resistance and inductance respectively. The symmetrical component voltages at the load are thus given by (17).

$$\left.\begin{array}{l} v^+(t) = e^+(t) - \Delta v^+(t) \\ v^-(t) = -\Delta v^-(t) \\ v^o(t) = -\Delta v^o(t) \end{array}\right\} \tag{17}$$

Appendix A shows that in general the sequence voltages at the load terminal can be written as follows;

$$\left.\begin{array}{l} v^+(t) = \hat{V}^+ \cos(\omega_1 t + \alpha^+) \\ v^-(t) = \hat{V}^- \cos(\omega_1 t + \alpha^-) \\ v^o(t) = \hat{V}^o \cos(\omega_1 t + \alpha^o) \end{array}\right\} \tag{18}$$

The magnitudes and phase angles are given in Appendix A. Considering the above equations, the modified phase voltages at the load in terms of the symmetrical components are determined by phase shifting the sequence voltages by −90°. Thus;

$$v_{mk}(t) = v_{mk}^+(t) + v_{mk}^-(t) + v_{mk}^o(t)\Big|_{k=a,b,c} = \hat{V}^+ e^{j(\omega_1 t + \alpha_k^+)} - \hat{V}^- e^{j(\omega_1 t + \alpha_k^-)} - \hat{V}^o e^{j(\omega_1 t + \alpha_k^o)}\Big|_{k=a,b,c} \tag{19}$$

Where;

$$\left.\begin{array}{l} \alpha_a^+ = \alpha^+,\ \alpha_b^+ = \alpha^+ - 120°,\ \alpha_c^+ = \alpha^+ + 120° \\ \alpha_a^- = \alpha_\Delta^-,\ \alpha_b^- = \alpha_\Delta^- + 120°,\ \alpha_c^- = \alpha_\Delta^- - 120° \\ \alpha_a^o = \alpha_\Delta^o,\ \alpha_b^o = \alpha_\Delta^o,\ \alpha_c^o = \alpha_\Delta^o \end{array}\right\} \tag{20}$$

The total IP is given by (21).

$$p(t) = \sum_{k=a,b,c} v_{mk}(t) i_k^*(t) \tag{21}$$

Equation (21) can be written in terms of the symmetrical components as follows;

$$p(t)=\begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix} v_{ma}^{+} & v_{mb}^{+} & v_{mc}^{+} \\ v_{ma}^{-} & v_{mb}^{-} & v_{mc}^{-} \\ v_{ma}^{o} & v_{mb}^{o} & v_{mc}^{o} \end{bmatrix}\begin{bmatrix} i_{a}^{+} & i_{a}^{-} & i_{a}^{o} \\ i_{b}^{+} & i_{b}^{-} & i_{b}^{o} \\ i_{c}^{+} & i_{c}^{-} & i_{c}^{o} \end{bmatrix}^{*}\begin{bmatrix}1\\1\\1\end{bmatrix} \tag{22.a}$$

$$p(t)=\begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix} p^{+} & p^{\pm} & p^{\overset{+}{o}} \\ p^{\mp} & p^{-} & p^{\overset{-}{o}} \\ p^{\overset{o}{+}} & p^{\overset{o}{-}} & p^{o} \end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} \tag{22.b}$$

Note that for simplicity, *(t)* has been eliminated from the equations, but it should be emphasised that all matrix elements are functions of time. The matrix elements are given below;

$$p^{+}=\sum_{k=a,b,c} v_{mk}^{+} i_{k}^{+*}=3V^{+}I^{+}e^{j\varphi^{+}} \tag{23}$$

$$p^{-}=\sum_{k=a,b,c} v_{mk}^{-} i_{k}^{-*}=-3V^{-}I^{-}e^{j\varphi^{-}} \tag{24}$$

$$p^{o}=\sum_{k=a,b,c} v_{mk}^{o} i_{k}^{o*}=-3V^{o}I^{o}e^{j\varphi^{o}}-3V^{o}I^{o}e^{j(2\omega_1 t+\lambda^{o})} \tag{25}$$

$$p^{\pm}=\sum_{k=a,b,c} v_{mk}^{+} i_{k}^{-*}=3V^{+}I^{-}e^{j(2\omega_1 t+\lambda^{\pm})} \tag{26}$$

$$p^{\overset{+}{o}}=\sum_{k=a,b,c} v_{mk}^{+} i_{k}^{o*}=0 \tag{27}$$

$$p^{\mp}=\sum_{k=a,b,c} v_{mk}^{-} i_{k}^{+*}=-3V^{-}I^{+}e^{j(2\omega_1 t+\lambda^{\mp})} \tag{28}$$

$$p^{\overset{-}{o}}=\sum_{k=a,b,c} v_{mk}^{-} i_{k}^{o*}=0 \tag{29}$$

$$p^{\overset{o}{+}}=\sum_{k=a,b,c} v_{mk}^{o} i_{k}^{+*}=0 \tag{30}$$

$$p^{\overset{o}{-}}=\sum_{k=a,b,c} v_{mk}^{o} i_{k}^{-*}=0 \tag{31}$$

Where V^{+}, V^{-} and V^{o} are the rms of pps, nps and zps voltages and I^{o} is the rms of zps current, and for the above;

$\varphi^{+}=\alpha^{+}-\beta^{+}$, $\varphi^{-}=\alpha_{\Delta}^{-}-\beta^{-}$, $\varphi^{o}=\alpha_{\Delta}^{o}-\beta^{o}\}$ (32.a)

$\lambda^{o}=\alpha_{\Delta}^{o}+\beta^{o}$, $\lambda^{\pm}=\alpha^{+}+\beta^{-}$, $\lambda^{\mp}=\alpha_{\Delta}^{-}+\beta^{+}\}$ (32.b)

The following remarks can be made;

1- The products of zps (in-phase) voltages and currents to pps and nps (balanced) currents and voltages are zero.
2- It can been seen that the product of the same component voltages and currents contains dc terms which it implies that there are active and reactive powers associated with these components.
3- In general, a sign cannot be defined for ac currents on their own, but when the flow of power, e.g. with respect to a voltage is considered the direction finds a meaning. As can be seen the sign of p^{+} and $p^{\pm}$ are positive. These terms are the product of the pps voltage to the pps and nps currents respectively. The same result was obtained in Section 2.1 when the source was considered.
4- The sign of p^{-}, p^{o}, and $p^{\mp}$ are negative compared to p^{+} and $p^{\pm}$. This means that these terms are flowing in opposite direction to p^{+} and $p^{\pm}$ w.r.t the unbalanced load. In fact, these IPs are exactly equal to the system impedance demand of the instantaneous components of p^{-}, p^{o}, and $p^{\mp}$. The same result for p^{-}, p^{o} had been obtained in [1] however, it is shown here that the same argument is also applied to the IPs that are the result of the product of unlike sequence quantities, namely nps voltage and pps currents.
   The physical explanation for this phenomenon is that since, as a result of unbalance loading, nps and zps currents flow in the circuit then there would be demand for p^{-}, p^{o}, and $p^{\mp}$ from the system impedance. The source cannot produce these power components, therefore the unbalanced load converts some of its p^{+} and $p^{\pm}$

into other dc and ac components of the IP respectively. Note that the sums of each of these components as well as the total IP in the circuit are zero and thus they satisfy the rule of conservation of energy.

5- If there is a balanced load in parallel with the unbalanced load, then it can be easily proved that due to the system impedance some unbalanced current would flow into the balanced load [1]. Thus; this causes the power components p^-, p^o, and $p^{\mp}$ to be appeared in the IP of the balanced load. The unbalanced load will also supply these.

With reference to the above observations, the flow of power components can be shown as in Fig 1.

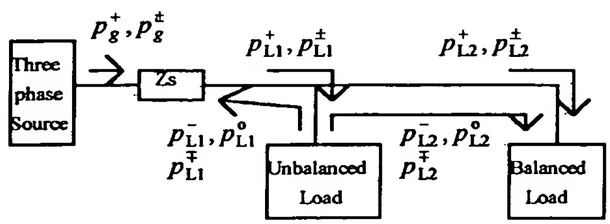

Fig 1- Flow of IP Components in a Unbalanced Circuit

Subscript L1 and L2 represent unbalanced and balanced loads respectively. Note that p^+ and $p^{\pm}$ demand by the system impedance is met by the source.

Hence, the balanced load also consumes (dissipate) some active power (real energy) in its nps and zps circuits. Although in some loads not all, like heating loads, these powers may be converted into useful energy but in general, due to the other detrimental effects of nps and zps currents on the system they are regarded as undesirable powers.

2.2.2-Total AP and pf at the Load

In general, considering balanced and unbalanced loads the total IP of the load can be written as follow;

$$p=p^+ + p^- + p^o + p^{\pm} + p^{\mp} \tag{33}$$

Substituting appropriate signal into (33) yields;

$$p=3\left(V^+I^+e^{j\varphi^+} + V^-I^-e^{j\varphi^-} + V^oI^oe^{j\varphi^o}\right) + 3\left[V^+I^-e^{j\lambda^{\pm}} + V^-I^+e^{j\lambda^{\mp}} + V^oI^oe^{j\lambda^o}\right]e^{j2\omega_1 t} \tag{34}$$

Note that, as previously mentioned, some of the terms in (34) would be negative for unbalanced loads.

As can be seen the IP consists of a complex dc and a complex ac term. The frequency of the ac component is double the system frequency as expected. The dc term that represents P and Q are the result of the product of the same symmetrical components.

The apparent power is defined as 2-Norm of the IP. Substituting (34) into (12) yields;

$$S=\left[\left|3V^+I^+e^{j\varphi^+} + 3V^-I^-e^{j\varphi^-} + 3V^oI^oe^{j\varphi^o}\right|^2 + \left|3V^+I^-e^{j\lambda^{\pm}} + 3V^-I^+e^{j\lambda^{\mp}} + 3V^oI^oe^{j\lambda^o}\right|^2\right]^{0.5} \tag{35}$$

or

$$S=\sqrt{P_t^2+Q_t^2+9\left[\left(V^+I^-\cos\lambda^{\pm} + V^-I^+\cos\lambda^{\mp} + V^oI^o\cos\lambda^o\right)^2 + \left(V^+I^-\sin\lambda^{\pm} + V^-I^+\sin\lambda^{\mp} + V^oI^o\sin\lambda^o\right)^2\right]} \tag{36}$$

Where: $P_t = P^+ + P^- + P^o$, $Q_t = Q^+ + Q^- + Q^o$,

$P^+ = 3V^+I^+\cos\varphi^+$, $Q^+ = 3V^+I^+\sin\varphi^+$

$P^- = 3V^-I^-\cos\varphi^-$, $Q^- = 3V^-I^-\sin\varphi^-$

$P^o = 3V^oI^o\cos\varphi^o$, $Q^o = 3V^oI^o\sin\varphi^o$

The AP defined by (36) can be written in terms of S, and S_u as given below;

$$S=\left\{S_s^2+S_u^2\right\}^{\frac{1}{2}} \tag{37}$$

Where

$$S_s=3\sqrt{\sum_{r=+,-,o}\left(V^rI^r\right)^2+\sum_{\substack{r1=+,-,o\\ r2=+,-,o\\ r1\neq r2}}V^{r1}V^{r2}I^{r1}I^{r2}\cos(\varphi^{r1}-\varphi^{r2})} \tag{38.a}$$

$$S_u=3\Big[\left(V^+I^-\right)^2+\left(V^-I^+\right)^2+\left(V^oI^o\right)^2+ 2V^+V^-I^+I^-\cos(\lambda^{\pm}-\lambda^{\mp})+2V^+V^oI^-I^o\cos(\lambda^{\pm}-\lambda^{o})+2V^-V^oI^+I^o\cos(\lambda^{o}-\lambda^{\mp})\Big]^{\frac{1}{2}} \tag{38.b}$$

Equation (38.a) and (38.b), which are in terms of symmetrical components, are equal to (45.a) and (45.b) in [2] respectively.
The pf defined by P/S can be deduced from (39).

$$pf=\frac{P_t}{S}=\frac{P^++P^-+P^o}{\left\{S_s^2+S_u^2\right\}^{0.5}} \tag{39}$$

Power factor specified by (39) is exactly the same as what is calculated from the phase quantities, obtained in [2].
As was previously mentioned, the unbalanced load converts some of its p^+ and $p^{\pm}$ into other IP components. This implies that P^-, P^o, Q^-, Q^o from dc terms and also $V^-I^+\exp(j\lambda^{\mp})$, $V^oI^o\exp(j\lambda^o)$ of the ac components in (35) and (36) would have opposite sign to P^+, Q^+ and $V^+I^-\exp(j\lambda^{\pm})$. This means that former quantities are actually flowing from the load towards the system and other balanced loads. The power that is actually consumed by the unbalanced load, P_t, is smaller than what is taken from the system. Although the difference is injected back to the system to be used by the system impedance and other balanced loads but the unbalanced load itself is the cause of the discrepancies between power components. The same argument is equally applicable to the calculated AP specified by (37).
Considering pf shown by (39), it can be seen that smaller S implies larger pf, however, P_t also would be smaller. Then some of increase in the pf due to decrease in S may be compensated by the lower value of, P_t. On the other hand, consumers pay their energy bill on active power as well as pf. Therefore, if P_t is considered for energy bills then the unbalanced loads do not pay for the system pollution that they are causing. It must also be said that these phenomenon is a mutual effect between loads and systems since the amount of power that is injected back to the system by the unbalanced load depends on the system impedance as well as amount of unbalanced current. Thus, from the consumer viewpoint, one may argue that the consumers must not alone be penalised for the pollution. Of course any supply system has some impedance which should comply with the standards. It is believed that if the supply system condition complies with the deign standards then the consumers should pay for the extra losses of the system.

3-MODIFIED APPARENT POWER AND POWER FACTOR

With reference to the above discussion it is desirable to use $P^+=3V^+I^+\cos\varphi^+$, for energy billing purposes. In this case the unbalanced load would pay for the pollution it is causing. On the other hand, balanced loads that are in parallel with unbalanced loads do not pay for the powers in the form of nps and zps active powers that they may be using. However, as was mentioned before, these powers are regarded as undesirable powers. It can be said that the balanced loads are forced to use these undesirable powers. Therefore, since the supply authorities receive the cost of energy from the source of pollution, there is no point of charging the balanced loads again.
If P^+ is used for energy metering then it is reasonable to suggest that the AP that is calculated from the pps voltage be considered for measuring the pf. This is different to what has been suggested in [1] as the AP used in pf calculation would be different. For unbalanced loads, P^+ tends to increase the pf but the AP calculated using only the pps voltages may be higher than the case when total AP is considered. Hence, higher S may compensate the increase in pf due to higher P^+.
Thus, the IP is calculated from (40.a);

$$p=v_{ma}^+i_a^*+v_{mb}^+i_b^*+v_{mc}^+i_c^* \tag{40.a}$$

$$p=3V^+I^+e^{j\varphi^+}+3V^+I^-e^{j\lambda^{\pm}}e^{j2\omega_1 t} \tag{40.b}$$

Where i_a, i_b and i_c are the phase currents and v^+_{ma}, v^+_{mb} and v^+_{mc} are the complex pps voltages. Note that the, (t), has been ignored for simplicity.
The AP is the 2-Norm of Equation (40.b). Thus;

$$S=\left[\left|3V^{+}I^{+}e^{j\varphi^{+}}\right|^{2}+\left|3V^{+}I^{-}e^{j\lambda^{\pm}}\right|^{2}\right]^{\frac{1}{2}} \tag{41}$$

Equation (41) can be written as below;

$$S=3\sqrt{\left(P^{+2}+Q^{+2}\right)+\left(V^{+}I^{-}\right)^{2}}=3V^{+}\sqrt{I^{+2}+I^{-2}} \tag{42}$$

Note that (42) has the same format as the AP at the source given by (13). The pf is thus given by (43).

$$pf=\frac{P^{+}}{S}=\frac{3V^{+}I^{+}\cos\varphi^{+}}{3V^{+}\sqrt{I^{+2}+I^{-2}}}=\frac{I^{+}\cos\varphi^{+}}{\sqrt{I^{+2}+I^{-2}}} \tag{43}$$

4-IMPLEMENTATION OF THE ALGORITHM

The AP can be calculated/measured either in time domain using (40.a) or from the product of the rms of pps voltage and pps and nps currents defined by (42). Note that since pps voltage is used in S calculation then the reference voltage is eliminated from the equation. Thus the AP given by (42) is independent of reference voltage. Both methods are described here.

4.1-Time Domain Technique

The average value of (40.b) is the pps active and reactive powers. In order to obtain (40.b), the pps voltages for three phases must be extracted from the phase signals. There are different methods that can be implemented to achieve this [3]. The description of these methods is out of the scope of the present work. Assuming that v^{+}_{a}, v^{+}_{b} and v^{+}_{c} are obtained then the quadrature voltages that are formed by phase shifting the pps voltages are obtained for each phase. Multiplication of the real and imaginary voltages with appropriate phase currents provides Equation (40.b). Thus;

$$v^{+}_{pk}=V^{+}\cos(\omega_1 t+\alpha^{+}_{k})\Big|_{k=a,b,c} \tag{44.a}$$

$$v^{+}_{qk}=V^{+}\sin(\omega_1 t+\alpha^{+}_{k})\Big|_{k=a,b,c} \tag{44.b}$$

Where;

$$\alpha^{+}_{a}=\alpha^{+},\ \alpha^{+}_{b}=\alpha^{+}-120^{\circ},\ \alpha^{+}_{b}=\alpha^{+}+120^{\circ}$$

$$p_p=\sum_{k=a,b,c} v^{+}_{pk} i_k=3V^{+}I^{+}\cos\varphi^{+}+3V^{+}I^{-}\cos(2\omega_1 t+\lambda^{\pm}) \tag{45.a}$$

$$p_q=\sum_{k=a,b,c} v^{+}_{qk} i_k=3V^{+}I^{+}\sin\varphi^{+}+3V^{+}I^{-}\sin(2\omega_1 t+\lambda^{\pm}) \tag{45.b}$$

As can be seen the average value of p_p and p_q are the pps active and reactive powers as given below;

$$P^{+}=\frac{1}{T}\int_{t}^{t+T} p_p\,dt \tag{46.a}$$

$$Q^{+}=\frac{1}{T}\int_{t}^{t+T} p_q\,dt \tag{46.a}$$

The total IP is given by (47) and the AP, defined as the rms of the IP is given by (48).

$$p=p_p+j\,p_q \tag{47}$$

$$S=\mathrm{rms}[p]=\left\{[\mathrm{rms}(p_p)]^{2}+[\mathrm{rms}(p_q)]^{2}\right\}^{\frac{1}{2}} \tag{48}$$

The components of S given by (48) are calculated as below;

$$\mathrm{rms}[p_p]=\sqrt{\frac{1}{T}\int_t^{t+T}p_p^2\,dt}=3V^+\sqrt{\left(I^+\cos\varphi^+\right)^2+\frac{I^{-^2}}{2}} \tag{49.a}$$

$$\mathrm{rms}[p_q]=\sqrt{\frac{1}{T}\int_t^{t+T}p_q^2\,dt}=3V^+\sqrt{\left(I^+\sin\varphi^+\right)^2+\frac{I^{-^2}}{2}} \tag{49.b}$$

By substituting (49.a) and (49.b) into (48) Equation (42) is deduced.

4.2-AP Calculation Using rms Values of Voltages and Current

Equation (42) can be determined from the rms values of the voltage and currents. It can be proved that the pps voltage is independent of reference voltage.

$$v_{\mathrm{meas\,k}}=v_k-v_{\mathrm{ref}}\Big|_{k=a,b,c} \tag{50}$$

$$3v^+=v_{\mathrm{meas\,a}}+Hv_{\mathrm{meas\,b}}+H^2v_{\mathrm{meas\,c}} \tag{51}$$

Where: $H=e^{j\frac{2\pi}{3}}$, $H^2=e^{-j\frac{2\pi}{3}}$

$v_{\mathrm{meas\,k}}$ = measured phase voltage

v_k = phase voltage

Thus

$$3v^+=v_a+Hv_b+H^2v_c-(1+H+H^2)v_{\mathrm{ref}} \tag{52}$$

Since ($1+H+H^2$)=0 then the reference voltage is eliminated from (52). The rms of the pps voltage is given by (53).

$$3V^+=\sqrt{\frac{1}{T}\int_t^{t+T}(3v^+)^2\,dt} \tag{53}$$

The amount of hardware and software for calculating the AP will be reduced if (42) can be arranged in terms of the phase currents. Equation (42) can be written as (54);

$$S=3V^+\sqrt{I^{+^2}+I^{-^2}}=3V^+\sqrt{I^{+^2}+I^{-^2}+I^{o^2}-I^{o^2}} \tag{54}$$

It can be shown that the following relationship exists between the rms values of phase and symmetrical component currents [1].

$$I_a^2+I_b^2+I_c^2=3\left(I^{+^2}+I^{-^2}+I^{o^2}\right) \tag{55}$$

Also the zps current is obtained by adding the phase currents. Hence, the zps current and its rms can be deduced from (56.a) and (56.b).

$$i^o=\frac{1}{3}(i_a+i_b+i_c) \tag{56.a}$$

$$I^o=\sqrt{\frac{1}{T}\int_t^{t+T}(i^o)^2\,dt} \tag{56.b}$$

Substituting (55) and (56.b) into (54) yields;

$$S=3V^+\sqrt{\frac{1}{3}\left(I_a^2+I_b^2+I_c^2\right)-I^{o^2}} \tag{57}$$

Where: $$I_k=\sqrt{\frac{1}{T}\int_t^{t+T}i_k^2\,dt}\;\Bigg|_{k=a,b,c} \tag{58}$$

5-CONCLUSIONS

The new AP described in [2] was represented in terms of the symmetrical components. The active and reactive powers were determined as dc signals on the real and imaginary axis. As in [2] the quantities defined as symmetrical and unsymmetrical AP were obtained in terms of the symmetrical components.

It was shown that the IP of 3-phase ideal source consists of a complex dc signal, which gives the active and reactive powers and is the result of the product of the pps voltage and current, and an ac signal which is the result of the product of pps voltage and nps current. The ac term is responsible for the increase in the AP when the system is unbalanced.

The reactive power is a physical quantity. Whenever there is a phase difference between the voltage and current, there exists a reactive power in the circuit. The difference between S and P not only depends on the existence of the reactive power but on the ac term of the IP. It may not be possible to attach any physical meaning to this term. However, it can be said that the ac term causes the increase in size of S.

The compensation of reactive power by passive elements also affects the ac components of the IP.

It was proved that unbalanced loads convert some of the power components into others and injects them back to the system to meet the demand by the system impedance and other parallel loads. As a result, it was suggested that only the products of the pps voltage with pps and nps currents be considered at the load. Also since the above argument is valid for the active and reactive powers then it was suggested that the pps active power is considered for the energy bill proposes. It was suggested that the pf is calculated from the pps power and the AP which is the result of the product of the pps voltage and pps and nps currents.

Two methods were presented for devising the algorithm using rms of the IP or rms of the pps voltage and appropriate currents.

6-REFERENCES

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7-APPENDIX A

Assume that the pps voltage generated by the source is given by (A1).

$$e(t)=\hat{E}\cos(\omega_1 t) \tag{A1}$$

The pps, nps and zps voltages at the load terminal are given by (A2).

$$\left.\begin{aligned} v^+(t)&=e^+(t)-\Delta v^+(t)\\ v^-(t)&=-\Delta v^-(t)\\ v^o(t)&=-\Delta v^o(t)\end{aligned}\right\} \tag{A2}$$

Where;

$$v^+(t)=\hat{V}^+\cos(\omega_1 t+\alpha^+) \tag{A3}$$

$$v^-(t)=\hat{V}^-\cos(\omega_1 t+\alpha^-) \tag{A4}$$

$$v^o(t)=\hat{V}^o\cos(\omega_1 t+\alpha^o) \tag{A5}$$

The magnitude and phase of each component are given below;

$$\hat{V}^+=\sqrt{\left[\hat{E}-Z_s^+\hat{I}^+\cos(\varphi_s^++\beta^+)\right]^2+\left[Z_s^+\hat{I}^+\sin(\varphi_s^++\beta^+)\right]^2} \tag{A6.a}$$

$$\alpha^+=\tan^{-1}\left[-\frac{Z_s^+\hat{I}^+\sin(\varphi_s^++\beta^+)}{\hat{E}-Z_s^+\hat{I}^+\cos(\varphi_s^++\beta^+)}\right] \tag{A6.b}$$

$$\bar{Z}_s^r=Z_s^r\angle\varphi_s^r\Big|_{r=+,-,o} \tag{A7}$$

$$Z_s^+=\sqrt{R_s^{+2}+(\omega_1 L_s^+)^2} \tag{A8.a}$$

$$\varphi_s^+=\tan^{-1}\left(\frac{\omega_1 L_s^+}{R_s^+}\right) \tag{A8.b}$$

$$\hat{V}^-=Z_s^-\hat{I}^-=\sqrt{R_s^{-2}+(\omega_1 L_s^-)^2}\;\hat{I}^- \tag{A9.a}$$

$$\alpha_{\Delta}^{-} = \varphi_s^{-} + \beta^{-} \quad \text{(A9.b)}$$

$$\hat{V}^{o} = Z_s^{o}\hat{I}^{o} = \sqrt{R_s^{o^2} + \left(\omega_1 L_s^{o}\right)^2}\;\hat{I}^{o} \quad \text{(A10.a)}$$

$$\alpha_{\Delta}^{o} = \varphi_s^{o} + \beta^{o} \quad \text{(A10.b)}$$

$$\alpha^{-} = \alpha_{\Delta}^{-} + \pi \quad \text{(A11)}$$

$$\alpha^{o} = \alpha_{\Delta}^{o} + \pi \quad \text{(A12)}$$

APPENDIX C

A NEW CONCEPT IN AC POWER THEORY

Part Three: The New Apparent Power and P wer factor with Non-Sinusoidal Wavef rms

F.Ghassemi

ABSTRACT In this paper the new power theory is investigated for non-sinusoidal waveforms. It will be shown that the instantaneous power components can be determined in frequency and time domains. The reactive power is determined as a dc imaginary signal in the form of the sum of harmonic reactive powers. This is equivalent to the Budeanu reactive power.
The apparent power in 1-phase and 3-phase systems will be investigated. The symmetrical components are used to identify the components of the instantaneous and apparent powers. It will be shown that the use of the fundamental frequency component is sufficient for calculating the active and apparent powers and thus the power factor. A new definition for the Distortion Power will be presented.

1-INTRODUCTION

The definition of active, reactive and apparent powers and also power factor (pf) when the waveforms are not sinusoidal have been investigated for many years. There is many literature on the subject that dates back as far as 1927 [1, 2, 3]. Ref. [1] highlights the reasons for using the apparent power in power engineering. For detailed study of the problem the reader is referred to the enormous literature on the subject.
In the first two parts of this report a new apparent power (AP) and pf was defined and investigated for 1-phase and 3-phase systems. The purpose of the present work is to apply the new definition of the AP to a non-sinusoidal waveform situation. The basis of the proposed power theory is such that the active, reactive powers and AP can be deduced in time or frequency domain. In the new method a quadrature axis voltage is obtained by phase shifting the voltage signal by –90° to form a complex voltage[4]. This complex voltage is then used to calculate the instantaneous power (IP). If the voltage(s) and current(s) are non-sinusoidal, then the IP will consists of different frequency terms. It will be shown that these frequency components may increase the size of the IP that is defined by its 2-Norm. The format and significance of the average value of the IP is also highlighted.
It will be discussed that although it is possible to devise a signal processing system (filter) with a phase response of –90° for all harmonics of the fundamental frequency to obtain the quadrature axis voltages, but this is not an essential requirement. The use of the fundamental frequency component (ffc) voltage may be sufficient for calculating the AP and hence the use of complicated signal processing with a particular phase response is obviated.

2-SINGLE PHASE POWER COMPONENTS

Consider an ideal source that feeds a non-linear load via a transmission circuit. The non-linear load draws a non-sinusoidal current. The source and load terminals are considered separately.

2.1-Source Terminal.

The voltage at the source and the current in the circuit are given by (1) and (2) respectively:

$$e(t)=\hat{E}_1\cos(\omega_1 t) \tag{1}$$

$$i(t)=\sum_{h=1}^{\infty}\hat{I}_h\cos(\omega_h t+\beta_h) \tag{2}$$

Where β_h is the phase angle of the harmonic currents with respect to (w.r.t) the source voltage and h is the harmonic order. The source can only generate the ffc voltage. The quadrature axis voltage and the IP are given by (3) and (4) respectively.

$$e_q(t)=\hat{E}_1\sin(\omega_1 t) \tag{3}$$

$$p(t)=e_m(t)i^*(t)=\hat{E}_1 e^{j\omega_1 t}\left[\sum_{h=1}^{\infty}\hat{I}_h\cos(\omega_h t+\beta_h)\right]^* \tag{4}$$

Equation (4) can be rewritten as follows;

$$p(t)=E_1 I_1 e^{j\varphi_1}+E_1 I_1 e^{j(2\omega_1 t+\lambda_1)}+\sum_{h=2}^{\infty}\left[E_1 I_h e^{j(\omega_{1h}t+\varphi_{1h})}+E_1 I_h e^{j(\omega^{1h}t+\lambda_{1h})}\right] \tag{5}$$

Where: E_1=rms of the real ffc voltage
I_1=rms of the ffc current
φ_1=phase angle between ffc voltage and current $=-\beta_1$
$\lambda_1=+\beta_1$
h= harmonic order
I_h= rms of harmonic components
$\omega_{1h}=\omega_1-\omega_h$, $\omega^{1h}=\omega_1+\omega_h$ for $h\neq 1$
$\varphi_{1h}=-\beta_h$, $\lambda_{1h}=\beta_h$ for $h\neq 1$

It can be seen that the IP consists of dc and ac components. The average real and imaginary values represent the standard definition of active and reactive powers. It is evident that the source can only produce these components by ffc. This is physically acceptable as the machine is driven at the fundamental angular frequency and the energy conversion and voltage generation takes place at this frequency only. The product of other frequency components with ffc voltage cannot have any average value neither on real nor imaginary axis.
It can be also seen that the ac terms do not have equal frequencies. The product of the ffc voltage with the current harmonics (excluding the ffc current) leads to ac terms with distinct negative and positive frequencies that are denoted by ω_{1h} and ω^{1h} respectively. The product of the ffc voltage and current also result in an ac term with the double frequency. The AP, which is the 2-Norm (rms) of (5), is thus given by (6).

$$S=\sqrt{(E_1I_1)^2+(E_1I_1)^2+\sum_{h=2}^{\infty}(E_1I_h)^2+\sum_{h=2}^{\infty}(E_1I_h)^2}=\sqrt{2}E_1\sqrt{\sum_{h=1}^{\infty}I_h^2} \qquad (6)$$

Note that the ac terms of (5) cannot have equal frequencies.
The same result can be deduced in frequency domain. By performing the convolution on the frequency spectrum of the voltage and current, given by (7.a) and (7.b) respectively, the same result as calculating the Fourier transform of (5) is obtained.

$$E(f)=\hat{E}\delta(f-f_1) \qquad (7.a)$$
$$I(f)=\frac{1}{2}\left[\hat{I}_h e^{j\beta_h}\delta(f-f_h)+\hat{I}_h e^{-j\beta_h}\delta(f+f_h)\right] \qquad (7.b)$$

The frequency spectrum of (5) is shown in Fig. 1.

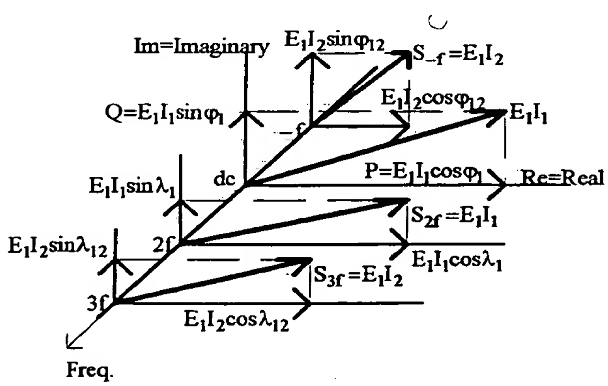

Fig. 1-Frequency Spectrum of the IP

It is clear that the AP given by (6) can also be calculated from the spectrum of the IP shown in Fig. 1.

$$S=\sqrt{(E_1I_1)^2+S_{2f}^2+S_{3f}^2+S_{-f}^2+\cdots} \qquad (8)$$

Where S_{2f}, S_{3f}, S_{-f}, ... are the magnitude of the ac-terms of the IP at different frequencies.
The term in bracket in (5) can be regarded as the Distortion Instantaneous Power (DIP), since it exists if the current is non-sinusoidal. Distortion Power (DP), which is a component of the AP, is defined as given in (9).

$$D=\sqrt{2}\sqrt{\sum_{h=2}^{\infty}(E_1I_h)^2} \qquad (9)$$

Thus, the AP can be rewritten as follows;

$$S=\sqrt{2P^2+2Q^2+D^2} \tag{10}$$

2.2-Load Terminal

The load terminal voltage is obtained by subtracting the voltage drop of the system impedance from the source voltage. Thus;

$$\Delta v_h(t)=(R_{sh}+L_{sh}\frac{d}{dt})\hat{I}_h\cos(\omega_h t+\beta_h) \tag{11.a}$$

$$\Delta v(t)=\sum_{h=1}^{\infty}\Delta v_h(t)=\sum_{h=1}^{\infty}\Delta\hat{V}_h\cos(\omega_h t+\alpha_{\Delta h}) \tag{11.b}$$

Where: R_{sh} =system impedance resistance at frequency h

L_{sh} =system impedance inductance at frequency h

$$\Delta\hat{V}_h=\sqrt{R_{sh}^2+(\omega_h L_{sh})^2}\,\hat{I}_h \tag{12.a}$$

$$\alpha_{\Delta h}=\beta_h+\varphi_{sh} \tag{12.b}$$

$$\varphi_{sh}=\tan^{-1}(\frac{\omega_h L_{sh}}{R_{sh}}) \tag{12.c}$$

$$v(t)=e(t)-\Delta v(t) \tag{13}$$

Substituting (11.b) into (13) yields;

$$v(t)=\hat{E}_1\cos(\omega_1 t)-\sum_{h=1}^{\infty}\Delta\hat{V}_h\cos(\omega_h t+\alpha_{\Delta h}) \tag{14.a}$$

$$v(t)=\hat{V}_1\cos(\omega_1 t+\alpha_1)+\sum_{h=2}^{\infty}\hat{V}_h\cos(\omega_h t+\alpha_h) \tag{14.b}$$

Where:

$$\hat{V}_1=\sqrt{[\hat{E}_1-\Delta\hat{V}_1\cos(\alpha_{\Delta 1})]^2+[\Delta\hat{V}_1\sin(\alpha_{\Delta 1})]^2} \tag{15.a}$$

$$\alpha_1=-\tan^{-1}[\frac{\Delta\hat{V}_1\sin(\alpha_{\Delta 1})}{\hat{E}_1-\Delta\hat{V}_1\cos(\alpha_{\Delta 1})}] \tag{15.b}$$

$$\hat{V}_h=\Delta\hat{V}_h\Big|_{h=2} \tag{16.a}$$

$$\alpha_h=\alpha_{\Delta h}+\pi\Big|_{h=2} \tag{16.b}$$

Equations (17.a) and (17.b) give the complex voltage signals for the system impedance and load terminal respectively.

$$\Delta v_m(t)=\sum_{h=1}^{\infty}\Delta\hat{V}_h e^{j(\omega_h t+\alpha_{\Delta h})} \tag{17.a}$$

$$v_m(t)=\sum_{h=1}^{\infty}\hat{V}_h e^{j(\omega_h t+\alpha_h)} \tag{17.b}$$

The corresponding IP are given below;

$$\Delta p=\Delta v_m(t)i^*(t)=\Delta\hat{V}_1\hat{I}_1 e^{j(\omega_1 t+\alpha_{\Delta 1})}\cos(\omega_1 t+\beta_1)+\sum_{h=2}^{\infty}\Delta\hat{V}_1\hat{I}_h e^{j(\omega_1 t+\alpha_{\Delta 1})}\cos(\omega_h t+\beta_h)$$
$$+\sum_{h=2}^{\infty}\Delta\hat{V}_h\hat{I}_h e^{j(\omega_h t+\alpha_{\Delta h})}\cos(\omega_h t+\beta_h)+\sum_{n=2}^{\infty}\sum_{\substack{m=1\\m\neq n}}^{\infty}\Delta\hat{V}_n\hat{I}_m e^{j(\omega_n t+\alpha_{\Delta n})}\cos(\omega_m t+\beta_n) \quad (18.a)$$

$$p=v_m(t)i^*(t)=\hat{V}_1\hat{I}_1 e^{j(\omega_1 t+\alpha_1)}\cos(\omega_1 t+\beta_1)+\sum_{h=2}^{\infty}\hat{V}_1\hat{I}_h e^{j(\omega_1 t+\alpha_1)}\cos(\omega_h t+\beta_h)$$
$$-\sum_{h=2}^{\infty}\Delta\hat{V}_h\hat{I}_h e^{j(\omega_h t+\alpha_{\Delta h})}\cos(\omega_h t+\beta_h)-\sum_{n=2}^{\infty}\sum_{\substack{m=1\\m\neq n}}^{\infty}\Delta\hat{V}_n\hat{I}_m e^{j(\omega_n t+\alpha_{\Delta n})}\cos(\omega_m t+\beta_n) \quad (18.b)$$

It can be seen that those terms of the IPs that do not involve the ffc voltage have equal magnitudes and opposite sign. This implies that the load actually supplies these components of the IP to the system impedance. For the system considered, the source of these components of the IP are the components that are the product of the ffc voltage and harmonic currents, namely;

$$p(t)=\hat{V}_1\hat{I}_1 e^{j(\omega_1 t+\alpha_1)}\cos(\omega_1 t+\beta_1)+\sum_{h=2}^{\infty}\hat{V}_1\hat{I}_h e^{j(\omega_1 t+\alpha_1)}\cos(\omega_h t+\beta_h) \quad (19)$$

(19) gives the IP components that the source can produce. Any other demand is met by the non-linear load by converting some of the IP components from (19). Note that the equivalent of (19) for the system impedance is also provided by the source. This phenomenon in non-sinusoidal situation is similar to the case discussed in [5], when 3-phase unbalanced system was considered. More discussion will be given later in this report when power factor (pf) is investigated.
(18.b) can be rearranged as follows;

$$p=\sum_{h=1}^{\infty}V_h I_h e^{j\varphi_h}+\sum_{h=1}^{\infty}V_h I_h e^{j(2\omega_h t+\lambda_h)}+\sum_{n=1}^{\infty}\sum_{\substack{m=1\\m\neq n}}^{\infty}V_n I_m e^{j(\omega_{nm}t+\varphi_{nm})}+\sum_{n=1}^{\infty}\sum_{\substack{m=1\\m\neq n}}^{\infty}V_n I_m e^{j(\omega^{nm}t+\lambda_{nm})} \quad (20)$$

Where: $\varphi_h=\alpha_h-\beta_h$, $\lambda_h=\alpha_h+\beta_h$, $\varphi_{nm}=\alpha_n-\beta_m$, $\lambda_{nm}=\alpha_n+\beta_m$
$\omega_{nm}=\omega_n-\omega_m$, $\omega^{nm}=\omega_n+\omega_m$
V and I with subscripts denote the rms of a specific harmonic.

(20) gives the total IP at the load terminal. Note that the average imaginary value of (20) is the Budeanu's reactive power. In that definition the total reactive power was defined as the sum of the magnitude of ac components with different frequencies. That was not acceptable to many researchers [6]. However, with the new method presented in this article, the reactive power is deduced as the sum of dc terms. This does not present any analytical problem. The real average value of (20) is the total active power consumed by the load. Thus;

$$P_t+jQ_t=\sum_{h=1}^{\infty}V_h I_h\cos\varphi_h+j\sum_{h=1}^{\infty}V_h I_h\sin\varphi_h \quad (21)$$

The frequency of the ac terms in (20), depends on the frequency spectra of the voltage and current. It is clear that the product of different frequency components of voltage and current in time domain (or convolution in frequency domain) may result in the formation of ac components with equal frequencies. Thus in general, combining the ac components with equal frequencies, the IP at the load terminal can be written as follows;

$$p=\sum_{h=1}^{\infty}V_h I_h e^{j\varphi_h}+\sum_{w=-\infty}^{\infty}S_w e^{j(\Omega_w t+\theta_w)} \quad (22)$$

Where: S_w= Magnitude of the ac terms for different frequencies
θ_w= Angle of the ac terms

The AP is thus given by (23).

$$S=\sqrt{\left(\sum_{h=1}^{\infty}V_h I_h\cos\varphi_h\right)^2+\left(\sum_{h=1}^{\infty}V_h I_h\sin\varphi_h\right)^2+\sum_{w=-\infty}^{\infty}S_w^2} \quad (23)$$

If the relationships given by (24) are valid for the IP then the AP can be written as (25).

$2\omega_h\neq\omega_{nm}$, $2\omega_h\neq\omega^{nm}$, $\omega_{nm}\neq\omega^{nm}$ } for any h, n and m (24)

$$S=\sqrt{\left(\sum_{h=1}^{\infty}V_h I_h\cos\varphi_h\right)^2+\left(\sum_{h=1}^{\infty}V_h I_h\sin\varphi_h\right)^2+\sum_{h=1}^{\infty}(V_h I_h)^2+2\sum_{n=1}^{\infty}\sum_{\substack{m=1\\ n\neq m}}^{\infty}(V_n I_m)^2} \tag{25}$$

The pf is equal to P/S. Thus using the total active power given by (21) and total AP described by (23), the pf can be calculated.

3-DISCUSSION

It was shown that the source can only produce those components of the IP that are the product of the ffc voltage and full spectrum of the current. Those terms of the system impedance IP that are the result of the product of the harmonic voltages (non-ffc voltage) and full spectrum of the current are supplied by the non-linear load. If the current spectrum of the non-linear load is not disturbed by any form of filtering, then the source of the above IP components are those terms that result in from the product of the ffc voltage and full spectrum of the current. If filters are used to modify the current spectrum then some of the non-ffc voltage components are produced by the filters.

Since the active power is only generated by the ffc voltages and currents then active harmonic powers are supplied by the non-linear load by converting some of the ffc power.

The above argument is also true for the harmonic power components of any linear load in parallel with the non-linear load that absorbs harmonic currents. In other words, the non-ffc voltage components of linear loads are supplied by the non-linear load. Note that the ffc active and reactive powers of the linear load are smaller than the total active and reactive powers as for linear loads the harmonic active and reactive powers are added to the ffc respective power components. However, harmonic powers have detrimental effects on the systems and one may argue that the linear loads are forced to consume these powers.

Hence, it is suggested that the IP is calculated only for ffc voltage components. The logic behind this is the fact that other power components are supplied by the ffc voltage terms. The non-linear load must be held responsible for the harmonic pollution, assuming the system impedance values comply with the design standards.

Thus the IP is calculated using (26).

$$p(t)=v_m(t)i^*(t)=V_1I_1e^{j\varphi_1}+V_1I_1e^{j(2\omega_1t+\lambda_1)}+\sum_{h=2}^{\infty}\left[V_1I_he^{j(\omega_{1h}t+\varphi_{1h})}+V_1I_he^{j(\omega^{1h}t+\lambda_{1h})}\right] \tag{26}$$

It can be seen that (26) has a format similar to the source IP. The ac terms do not have equal frequencies. The average real and imaginary values of (26) are the ffc active and reactive powers respectively. The AP and pf of the load are thus given by (27) and (28) respectively.

$$S=\sqrt{2}\sqrt{(V_1I_1)^2+\sum_{h=2}^{\infty}(V_1I_h)^2}=\sqrt{2}V_1\sqrt{\sum_{h=1}^{\infty}I_h^2} \tag{27}$$

$$pf=\frac{P}{S}=\frac{V_1I_1\cos\varphi_1}{\sqrt{2}V_1\sqrt{\sum_{h=1}^{\infty}I_h^2}}=\frac{I_1\cos\varphi_1}{\sqrt{2}\sqrt{\sum_{h=1}^{\infty}I_h^2}} \tag{28}$$

It can be said that the AP and pf given respectively by (27) and (28) reflect the electrical quality of the load in single phase pure/non sinusoidal situations.

4-IMPLEMENTATION OF THE TECHNIQUE

In order to calculate the AP, the voltage signal must be phase shifted by −90° to obtain the quadrature axis voltage signal. Thus, a system is required with a phase response of −90° for all frequencies in the bandwidth considered. However, it was shown that the product of the ffc voltage and full spectrum of the current can reflect the quality of the load. Hence, it is essential to filter the voltage signal to extract the ffc voltage and then to perform the phase shifting process. This eliminates the need for a sophisticated filtering and signal processing. Appendix A shows that (29) can be used to calculate the AP.

$$S=\sqrt{\left\{\mathrm{rms}\left[v_{1p}(t)i(t)\right]\right\}^2+\left\{\mathrm{rms}\left[v_{1q}(t)i(t)\right]\right\}^2} \tag{29}$$

Where: $v_{1p}(t)=\hat{V}_1\cos(\omega_1t+\alpha_1)$, $v_{1q}(t)=\hat{V}_1\sin(\omega_1t+\alpha_1)$

$i(t)$ is given by (2)

5-THREE PHASE SYSTEMS

The procedure for calculating the AP in 3-phase systems when the signals are not sinusoidal is exactly the same as the case that has been described in [4] and the proceeding sections.

5.1-AP Calculation Using Phase Quantities

Assume the phase voltages and currents at any point in the circuit are given below:

$$v_k(t)=\sum_{h=1}^{\infty}\hat{V}_{kh}\cos(\omega_h t+\alpha_{kh})\Big|_{k=a,b,c} \tag{30.a}$$

$$i_k(t)=\sum_{h=1}^{\infty}\hat{I}_{kh}\cos(\omega_h t+\beta_{kh})\Big|_{k=a,b,c} \tag{30.b}$$

Where k and h denote phase number and harmonic order respectively. Note that in order to consider a general case the harmonic order for different phases may not be the same.

The complex voltages and the total IP are determined from (31) and (32) respectively.

$$v_{mk}(t)=\sum_{h=1}^{\infty}\hat{V}_{kh}e^{j(\omega_h t+\alpha_{kh})}\Big|_{k=a,b,c} \tag{31}$$

$$p(t)=\sum_{h=1}^{\infty}\sum_{k=a,b,c}V_{kh}I_{kh}e^{j\varphi_{kh}}+\sum_{w=-\infty}^{\infty}\sum_{k=a,b,c}S_{kw}e^{j(\Omega_w t+\theta_{kw})} \tag{32}$$

Where V_{kh} and I_{kh} are the rms of the per phase harmonic voltages and currents. S_{kw} is the magnitude of the ac terms of the IP.

The AP is defined as the 2-Norm of (32). Thus:

$$S=\left\{\left(\sum_{h=1}^{\infty}\sum_{k=a,b,c}V_{kh}I_{kh}\cos\varphi_{kh}\right)^2+\left(\sum_{h=1}^{\infty}\sum_{k=a,b,c}V_{kh}I_{kh}\sin\varphi_{kh}\right)^2+\sum_{w=-\infty}^{\infty}\left[\left(\sum_{k=a,b,c}S_{kw}\cos\theta_{kh}\right)^2+\left(\sum_{k=a,b,c}S_{kw}\sin\theta_{kh}\right)^2\right]\right\}^{\frac{1}{2}} \tag{33}$$

S described by (33) is the total AP of the non-linear load itself, excluding the demand by the system impedance and any other linear parallel load that takes some of the harmonic currents. The pf is the ratio of the active power, that is the real dc value of (32), and S given by (33).

At the source and also if the load terminal ffc voltage is considered in the power analysis of 3-phase systems, then the IP can be calculated from (34).

$$p(t)=\sum_{k=a,b,c}V_{k1}I_{k1}e^{j\varphi_{k1}}+\sum_{k=a,b,c}V_{k1}I_{k1}e^{j(2\omega_1 t+\lambda_{k1})}+\sum_{k=a,b,c}\sum_{h=2}^{\infty}v_{k1}i^{*}_{kh}$$

$$p(t)=\sum_{k=a,b,c}V_{k1}I_{k1}e^{j\varphi_{k1}}+\sum_{k=a,b,c}V_{k1}I_{k1}e^{j(2\omega_1 t+\lambda_{k1})}+\sum_{h=2}^{\infty}\sum_{k=a,b,c}V_{k1}I_{kh}e^{j(\omega_{1h}t+\varphi_{k1h})}+\sum_{h=2}^{\infty}\sum_{k=a,b,c}V_{k1}I_{kh}e^{j(\omega^{1h}t+\lambda_{k1h})} \tag{34}$$

Where for the source V_{k1} is replaced by E_1. It can be seen that the ac part consists of terms with distinct positive and negative frequencies. The AP is thus given below:

$$S=\sqrt{S_s^2+S_u^2+D^2} \tag{35}$$

Where:

$$S_s=\left|\sum_{k=a,b,c}V_{k1}I_{k1}e^{j\varphi_{k1}}\right| \quad \text{As given in [4]} \tag{36.a}$$

$$S_u=\left|\sum_{k=a,b,c}V_{k1}I_{k1}e^{j\lambda_{k1}}\right| \quad \text{As given in [4]} \tag{36.b}$$

$$D=\sqrt{\sum_{h=2}^{\infty}\left|\sum_{k=a,b,c}V_{k1}I_{kh}e^{j\varphi_{k1h}}\right|^2+\sum_{h=2}^{\infty}\left|\sum_{k=a,b,c}V_{k1}I_{kh}e^{j\lambda_{k1h}}\right|^2} \tag{37}$$

Where D is the Distortion Power. Note that S_s and S_u are exactly the same as (45.a) and (45.b) in [4]. These terms are related to the ffc voltages and currents.

The pf is then calculated using real dc value of (34) and S given by (35).

5.2-AP Calculation Using Symmetrical Components

If the system impedance can be considered equal in all three phases, then the 3-phase system can be replaced by three uncoupled systems known as sequence impedances.

The above statement is very near to true in a 3-phase system when only ffc signals are present. However, if there is a non-linear load in the circuit, then mutual effects between sequence networks exist. This is due to the fact that the condition of system impedance symmetry may not be fulfilled since the self and mutual impedances of the phases may be different as the frequency increases.

However, it is always possible to resolve the voltages and currents at any point in the system to symmetrical components. The system impedance asymmetry only results in coupling between the sequence networks. This implies that the current in one sequence circuit gives rise to a voltage in others [7]. Therefore, having measured/calculated the phase co-ordinates voltages and currents then wide band frequency spectrum three phase signals can be resolved into symmetrical components. Unequal system impedances implies that Equation (16) in [5] is not strictly valid since voltage drop on sequence impedances of the line is not due to one sequence current only.

The voltages and currents are defined by (30.a) and (30.b). At any point in the circuit the following relationship between the phase and symmetrical component voltages exists.

$$v^+(t)=\sum_{h=1}^{\infty}v_h^+(t)=\frac{1}{3}\sum_{h=1}^{\infty}\left[v_{ah}(t)+v_{bh}(t+\frac{T_h}{3})+v_{ch}(t-\frac{T_h}{3})\right] \tag{38.a}$$

$$v^-(t)=\sum_{h=1}^{\infty}v_h^-(t)=\frac{1}{3}\sum_{h=1}^{\infty}\left[v_{ah}(t)+v_{bh}(t-\frac{T_h}{3})+v_{ch}(t+\frac{T_h}{3})\right] \tag{38.b}$$

$$v^o(t)=\sum_{h=1}^{\infty}v_h^o(t)=\frac{1}{3}\sum_{h=1}^{\infty}\left[v_{ah}(t)+v_{bh}(t)+v_{ch}(t)\right] \tag{38.c}$$

Where T_h is the period for each harmonic including the ffc. The same relationship can be defined for the currents.

If the 3-phase system is balanced, then harmonics will appear only in one symmetrical component signal. For example, for balanced 3-phase systems the 9th harmonic appears in zero phase sequence (zps) only and the 2nd harmonic as negative phase sequence (nps). However, considering a general case, the positive phase sequence (pps), nps and zps signals may contain harmonics irrespective of their orders.

The symmetrical component currents and complex voltages are defined by (39) and (40) respectively.

$$i_k^r(t)=\sum_{h=1}^{\infty}\hat{I}_{kh}^r\cos(\omega_h t+\beta_{kh}^r)\Bigg|^{r=+,-,o}\Bigg|_{k=a,b,c} \tag{39}$$

$$v_{mk}^r(t)=\sum_{h=1}^{\infty}v_{mkh}^r(t)=\sum_{h=1}^{\infty}\hat{V}_{kh}^r e^{j(\omega_h t+\alpha_{kh}^r)}\Bigg|^{r=+,-,o}\Bigg|_{k=a,b,c} \tag{40}$$

Where:

$$\begin{array}{lll}
\alpha_{ah}^+=\alpha_h^+ , & \alpha_{bh}^+=\alpha_h^+-120^\circ , & \alpha_{ch}^+=\alpha_h^++120^\circ \\
\alpha_{ah}^-=\alpha_h^- , & \alpha_{bh}^-=\alpha_h^-+120^\circ , & \alpha_{ch}^-=\alpha_h^--120^\circ \\
\alpha_{ah}^o=\alpha_h^o , & \alpha_{bh}^o=\alpha_h^o , & \alpha_{ch}^o=\alpha_h^o \\
\beta_{ah}^+=\beta_h^+ , & \beta_{bh}^+=\beta_h^+-120^\circ , & \beta_{ch}^+=\beta_h^++120^\circ \\
\beta_{ah}^-=\beta_h^- , & \beta_{bh}^-=\beta_h^-+120^\circ , & \beta_{ch}^-=\beta_h^--120^\circ \\
\beta_{ah}^o=\beta_h^o , & \beta_{bh}^o=\beta_h^o , & \beta_{ch}^o=\beta_h^o
\end{array}$$

The total IP is defined by (22.a) or (22.b) in [5]. The elements of the total IP are given below;

$$p^+=\sum_{k=a,b,c}v_{mk}^+(t)i_k^{+*}(t)=3\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}V_n^+I_m^+e^{j(\omega_{nm}t+\varphi_{nm}^+)}=3\sum_{h=1}^{\infty}V_h^+I_h^+e^{j\varphi_h^+}+3\sum_{n=1}^{\infty}\sum_{\substack{m=1\\m\neq n}}^{\infty}V_n^+I_m^+e^{j(\omega_{nm}t+\varphi_{nm}^+)} \tag{41}$$

$$p^-=\sum_{k=a,b,c}v_{mk}^-(t)i_k^{-*}(t)=3\sum_{h=1}^{\infty}V_h^-I_h^-e^{j\varphi_h^-}+3\sum_{n=1}^{\infty}\sum_{\substack{m=1\\m\neq n}}^{\infty}V_n^-I_m^-e^{j(\omega_{nm}t+\varphi_{nm}^-)} \tag{42}$$

$$p^{o}=\sum_{k=a,b,c} v_{mk}^{o}(t) i_{k}^{o*}(t)=3\sum_{h=1}^{\infty} V_h^o I_h^o e^{j\varphi_h^o}+3\sum_{n=1}^{\infty}\sum_{\substack{m=1\\m\neq n}}^{\infty} V_n^o I_m^o e^{j(\omega_{nm}t+\varphi_{nm}^o)}+3\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} V_n^o I_m^o e^{j(\omega^{nm}t+\lambda_{nm}^o)} \tag{43}$$

$$p^{\pm}=\sum_{k=a,b,c} v_{mk}^{+}(t) i_{k}^{-*}(t)=3\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} V_n^+ I_m^- e^{j(\omega^{nm}t+\lambda_{nm}^{\pm})} \tag{44}$$

$$p^{\mp}=\sum_{k=a,b,c} v_{mk}^{-}(t) i_{k}^{+*}(t)=3\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} V_n^- I_m^+ e^{j(\omega^{nm}t+\lambda_{nm}^{\mp})} \tag{45}$$

$$\sum_{k=a,b,c} v_{mk}^{+} i_k^{o*}=\sum_{k=a,b,c} v_{mk}^{-} i_k^{o*}=\sum_{k=a,b,c} v_{mk}^{o} i_k^{+*}=\sum_{k=a,b,c} v_{mk}^{o} i_k^{-*}=0 \tag{46}$$

Where: $\omega_{nm}=\omega_n-\omega_m$ $\quad\omega^{nm}=\omega_n+\omega_m$

$\varphi_{nm}{}^{r}=\alpha_n{}^{r}-\beta_m{}^{r}$ $\quad\lambda_{nm}{}^{r}=\alpha_n{}^{r}+\beta_m{}^{r}$

h, n, m = harmonic orders.

With reference to [5] and the aforementioned analysis the following remarks can be made.

1- The product of similar sequences and harmonics leads to dc values in p^+, p^- and p^o.
2- If the signals are pure sinusoidal, the result is the same as what was obtained in [5].
3- The ac components of p^+ and p^- have frequencies that result in from the difference of the harmonic frequencies.
4- The ac components of p^o have frequencies that are from the difference and summation of the harmonic frequencies.
5- $p^{\pm}$ and $p^{\mp}$ have frequencies that result in from the summation of the harmonic frequencies.
6- At the ideal source terminal p^-, p^o and $p^{\mp}$ are all zero. The average value is $V_1{}^+I_1{}^+\exp(j\varphi_1{}^+)$.
7- For non-linear loads, p^-, p^o and $p^{\mp}$ at all frequencies are equal to the demand of the system impedance and any other parallel load that absorbs the harmonic and unbalanced currents of the non-linear load.
8- The terms mentioned in 7 are supplied through p^+ and $p^{\pm}$ by the source for the system considered. Note that if there is any form of harmonic filtering, then the above terms are supplied by the source and filter.

Combining the ac terms with equal frequencies, the total IP is given below:

$$p(t)=p^{+}+p^{-}+p^{o}+p^{\pm}+p^{\mp} \tag{47}$$

$$p(t)=3\sum_{r=+,-,o}\sum_{h=1}^{\infty} V_h^r I_h^r e^{j\varphi_h^r}+3\sum_{w=-\infty}^{\infty} S_w e^{j(\Omega_w t+\theta_w)} \tag{48}$$

The average values of (48) on real and imaginary axis are the load active and reactive powers respectively as shown below.

$$P=3\left[\sum_{h=1}^{\infty} V_h^+ I_h^+ \cos\varphi_h^+ +\sum_{h=1}^{\infty} V_h^- I_h^- \cos\varphi_h^- +\sum_{h=1}^{\infty} V_h^o I_h^o \cos\varphi_h^o\right] \tag{49}$$

$$Q=3\left[\sum_{h=1}^{\infty} V_h^+ I_h^+ \sin\varphi_h^+ +\sum_{h=1}^{\infty} V_h^- I_h^- \sin\varphi_h^- +\sum_{h=1}^{\infty} V_h^o I_h^o \sin\varphi_h^o\right] \tag{50}$$

(49) and (50) are equal to the real and imaginary average values of (32). The ac terms are also equal for each frequency. The AP in terms of the symmetrical components is the 2 − Norm of (48). Thus;

$$S=3\sqrt{\left|\sum_{r=+,-,o}\sum_{h=1}^{\infty} V_h^r I_h^r e^{j\varphi_h^r}\right|^2+\sum_{w=-\infty}^{\infty} S_w^2} \tag{51.a}$$

$$S=3\sqrt{\left(\sum_{r=+,-,o}\sum_{h=1}^{\infty} V_h^r I_h^r \cos\varphi_h^r\right)^2+\left(\sum_{r=+,-,o}\sum_{h=1}^{\infty} V_h^r I_h^r \sin\varphi_h^r\right)^2+\sum_{w=-\infty}^{\infty} S_w^2} \tag{51.b}$$

If one can assumes that (16) in [5] is approximately valid, i.e. the transmission system impedance matrix is symmetrical for all frequencies, then it is technically more acceptable to consider the ffc, pps voltage in the IP calculation. Thus, the IP is determined from (53).

$$p(t)=\sum_{k=a,b,c} v^{+}_{mk1}\left(\sum_{h=1}^{\infty} i^{+}_{kh}+\sum_{h=1}^{\infty} i^{-}_{kh}+\sum_{h=1}^{\infty} i^{o}_{kh}\right)^{*} \tag{52}$$

$$p(t)=3V_1^{+}I_1^{+}e^{j\varphi_1^{+}}+3V_1^{+}\sum_{h=2}^{\infty} I_h^{+}e^{j(\omega_{1h}t+\varphi_{1h}^{+})}+3V_1^{+}\sum_{h=1}^{\infty} I_h^{-}e^{j(\omega^{1h}t+\lambda_{1h}^{\pm})} \tag{53}$$

It can be seen that the ac terms do not have equal frequencies. The product of the ffc, pps voltage with pps currents leads to a complex average value and ac terms with distinct negative frequencies. The product of the ffc, pps voltage and the nps currents, on the other hand, results in the ac terms with distinct positive frequencies. The active, reactive and apparent powers are given by (54.a), (54.b) and (55.b) respectively.

$$P=3V_1^{+}I_1^{+}\cos\varphi_1^{+} \tag{54.a}$$

$$Q=3V_1^{+}I_1^{+}\sin\varphi_1^{+} \tag{54.b}$$

$$S=\sqrt{\left(3V_1^{+}I_1^{+}\right)^2+\sum_{h=2}^{\infty}\left(3V_1^{+}I_h^{+}\right)^2+\sum_{h=1}^{\infty}\left(3V_1^{+}I_h^{-}\right)^2} \tag{55.a}$$

$$S=3V_1^{+}\sqrt{\sum_{h=1}^{\infty} I_h^{+2}+\sum_{h=1}^{\infty} I_h^{-2}} \tag{55.b}$$

The AP in terms of S_s, S_u and DP can be written as follows;

$$S=\sqrt{S_s^2+S_u^2+D^2} \tag{56}$$

Where:

$$S_s=3V_1^{+}I_1^{+} \tag{57.a}$$

$$S_s=3V_1^{+}I_1^{-} \tag{57.b}$$

$$D=3V_1^{+}\sqrt{\sum_{h=2}^{\infty} I_h^{+2}+\sum_{h=2}^{\infty} I_h^{-2}} \tag{57.c}$$

It can be shown that the power components at the load terminal have the same format as the source where V_1^{+} is replaced by E_1.

The pf is calculated from P/S where P and S are given by (54.a) and (55.b).

With reference to [5] and since the relationship given by (58) is valid between phase and sequence currents at each frequency h then (55.b) can be written in terms of the phase and zps currents as given by (59).

$$I_h^{+2}+I_h^{-2}+I_h^{o2}=\frac{1}{3}\left(I_{ah}^2+I_{bh}^2+I_{ch}^2\right) \tag{58}$$

$$S=3V_1^{+}\sqrt{\frac{1}{3}\left(\sum_{h=1}^{\infty} I_{ah}^2+\sum_{h=1}^{\infty} I_{bh}^2+\sum_{h=1}^{\infty} I_{ch}^2\right)-\sum_{h=1}^{\infty} I_h^{o2}} \tag{59}$$

The DP can be written in terms of the phase and zps current as shown by (60).

$$D=3V_1^{+}\sqrt{\frac{1}{3}\left(\sum_{h=2}^{\infty} I_{ah}^2+\sum_{h=2}^{\infty} I_{bh}^2+\sum_{h=2}^{\infty} I_{ch}^2\right)-\sum_{h=2}^{\infty} I_h^{o2}} \tag{60}$$

6-THREE PHASE IMPLEMENTATION

The technique for measuring the AP in 3-phase systems is similar to the single phase method that has already been explained in Section 4. There is a need to extract the pps voltage after the filter section. The AP is then calculated according to [5]. It is essential to specify the bandwidth of the measuring units.

7-CONCLUSION

The new method for calculating the AP and pf was investigated when the signals are not sinusoidal. Single and three phase systems were considered. It was shown that the technique can be equally applied in frequency and time domains. The active and reactive powers are the sum of the active and reactive powers of each frequency and these are determined as the average powers of the total IP. This partly confirms the Budeanu definition f the reactive power. The difference in the two approaches is the fact that in the new definition the reactive power is determined as the sum of dc values.

It was shown that it is sufficient and technically more acceptable to consider the ffc voltage in the IP calculation at any point in the circuit. This reflects the power components that are present at the source.
With reference to [5], in 3-phase systems the pps, ffc voltage must be considered in the IP calculation.
The Distortion Power was defined in terms of the phase and sequence quantities.

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9-APPENDIX A

The voltage signal must be filtered to extract the fundamental frequency component (ffc). The filtered signal is then phase shifted to form the complex ffc voltage. Thus;

$$v_{1p}(t)=\hat{V}_1\cos(\omega_1 t+\alpha_1) \tag{A1.a}$$

$$v_{1q}(t)=\hat{V}_1\cos(\omega_1 t+\alpha_1-\frac{\pi}{2})=\hat{V}_1\sin(\omega_1 t+\alpha_1) \tag{A1.b}$$

$$v_m(t)=v_{1p}(t)+jv_{1q}(t) \tag{A2}$$

The IP is given by (A3).

$$p(t)=p_p(t)+jp_q(t) \tag{A3}$$

Where:

$$p_p(t)=v_{1p}(t)\sum_{h=1}^{\infty}i_h(t)=\hat{V}_1\cos(\omega_1 t+\alpha_1)\sum_{h=1}^{\infty}\hat{I}_h\cos(\omega_h t+\beta_h) \tag{A4}$$

$$p_q(t)=v_{1q}(t)\sum_{h=1}^{\infty}i_h(t)=\hat{V}_1\sin(\omega_1 t+\alpha_1)\sum_{h=1}^{\infty}\hat{I}_h\cos(\omega_h t+\beta_h) \tag{A5}$$

(A4) and (A5) can be written as follows;

$$p_p(t)=V_1I_1\cos\varphi_1+V_1I_1\cos(2\omega_1 t+\lambda_1)+\sum_{h=2}^{\infty}V_1I_h\cos(\omega_{1h}t+\varphi_{1h})+\sum_{h=2}^{\infty}V_1I_h\cos(\omega^{1h}t+\lambda_{1h}) \tag{A6}$$

$$p_q(t)=V_1I_1\sin\varphi_1+V_1I_1\sin(2\omega_1 t+\lambda_1)+\sum_{h=2}^{\infty}V_1I_h\sin(\omega_{1h}t+\varphi_{1h})+\sum_{h=2}^{\infty}V_1I_h\sin(\omega^{1h}t+\lambda_{1h}) \tag{A7}$$

Where; $\omega_{1h}=\omega_1-\omega_h$, $\omega^{1h}=\omega_1+\omega_h$

V and I with subscripts denote the rms of the signals.

The AP is defined as the rms of the IP. Hence;

$$S=\mathrm{rms}[p(t)]=\sqrt{\{\mathrm{rms}[p_p(t)]\}^2+\{\mathrm{rms}[p_q(t)]\}^2} \quad \text{(A8)}$$

Where:

$$\mathrm{rms}[p_p(t)]=\sqrt{(V_1I_1\cos\varphi_1)^2+\frac{(V_1I_1)^2}{2}+\sum_{h=2}^{\infty}\frac{(V_1I_h)^2}{2}+\sum_{h=2}^{\infty}\frac{(V_1I_h)^2}{2}} \quad \text{(A9)}$$

$$\mathrm{rms}[p_p(t)]=\sqrt{(V_1I_1\sin\varphi_1)^2+\frac{(V_1I_1)^2}{2}+\sum_{h=2}^{\infty}\frac{(V_1I_h)^2}{2}+\sum_{h=2}^{\infty}\frac{(V_1I_h)^2}{2}} \quad \text{(A10)}$$

$$S=\sqrt{2}\sqrt{(V_1I_1)^2+\sum_{h=2}^{\infty}(V_1I_h)^2}=\sqrt{2}V_1\sqrt{\sum_{h=1}^{\infty}I_h^2} \quad \text{(A11)}$$

The AP can be written in terms of active, reactive and distortion powers as shown in (A12).

$$S=\sqrt{2P^2+2Q^2+D^2} \quad \text{(A12)}$$

Where:

$$P=V_1I_1\cos\varphi_1=\mathrm{average}[p_p(t)] \quad \text{(A13.a)}$$

$$Q=V_1I_1\sin\varphi_1=\mathrm{average}[p_q(t)] \quad \text{(A13.b)}$$

$$D=\sqrt{\left\{\mathrm{rms}\left[v_{1p}(t)\sum_{h=2}^{\infty}i_h(t)\right]\right\}^2+\left\{\mathrm{rms}\left[v_{1q}(t)\sum_{h=2}^{\infty}i_h(t)\right]\right\}^2} \quad \text{(A14)}$$

APPENDIX D

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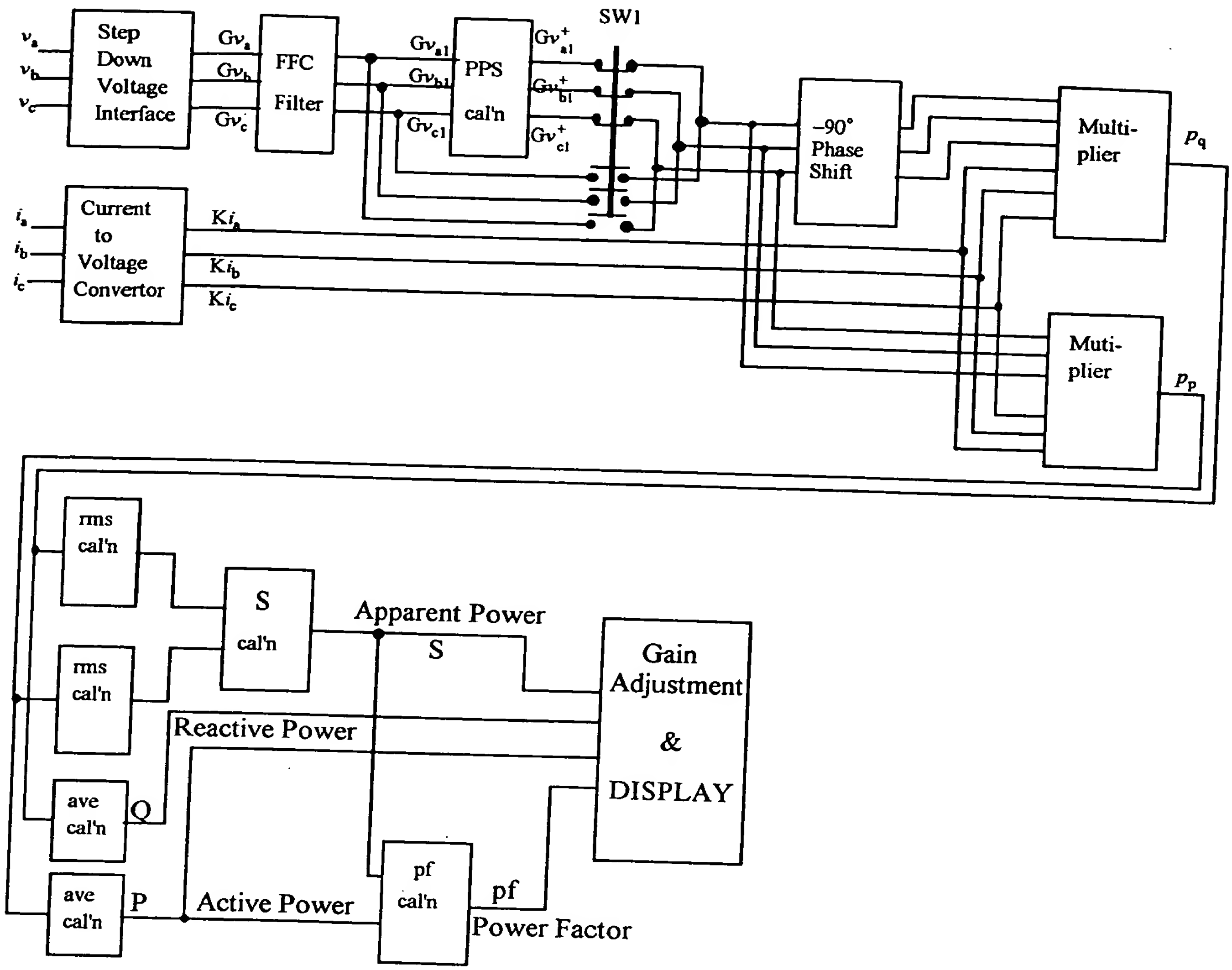

Figure 1- Schematic Diagram of the Power Meter

Key;

FFC: Fundamental Frequency Component
PPS: Positive Phase Sequence
cal'n.: calculation
rms: root mean squared
ave: average
● connection